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**PiXL Independence:**

**Mathematics** –Answer Booklet

KS4 HIGHER

**Topic 3 - Factorising, Inequalities, Quadratics**

**Contents:**

Answers

1. **Basic Skills Check**

*Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator where possible.*

**Skills Check 1**

1. A coat is reduced by 12% to a price of £59.84. Calculate the original cost of the coat.

 £68

1. What is $\frac{2 }{3}$ of £126?

 84

1. Write 58 as a product of its prime factors.

 2 x29

1. Factorise x² -6x -16.

$$ (x-8)(x+2)$$

1. Write 760,000,000 in standard form.

 7.6 x 108

1. Solve: $4x+7\leq -13$

$$x\leq -5$$

1. Use prime factors to find the lowest common multiple of 70 and 80.

 560

1. List the first 5 terms of the sequence -3n- 2.

-5, -8, -11, -14, -17

1. Find the total perimeter of the sector shown, correct to one decimal place.

26.1cm

135˚

6cm

1. Calculate , giving your answer in standard form correct to **two** significant figures. 3.0 x105

**Skills Check 2**

1. The height of a student is measured to the nearest cm, if it is recorded as 132cm what is the maximum and minimum height of the student? UB= 132.5 LB=131.5

2. Calculate , giving your answer in standard form correct to two significant figures.

 1.5 x1016

3. Write 40 as a product of prime factors. What is the LCM of 40 and 52?

$$ 2^{3}×5=40$$

$$ 2^{2}×13=52$$

 LCM = 520

4. Factorise x² -2x - 80

 (x-10) (x+8).

5. Write 0.00000302 in standard form.

 3.02 x10-6

6. Solve: $-17\leq 4-3x$

 $x\leq 7$

1. Calculate the total surface area of a cylinder with radius 3cm and length 11cm. Give your **final** answer to one decimal place.

Circle area = 18π

11cm

3cm

Curved surface = 66π

Total = 263.9cm2

1. Find the nth term of the sequence: -2, 1, 6, 13….. $n^{2}-3$
2. Find the reciprocal of the number 3.6, giving your answer as a fraction.

5/18

1. The masses of a group of pupils are displayed in this table. Calculate an estimate of the mean mass.

|  |  |  |  |
| --- | --- | --- | --- |
| Mass (*x* kg) | Frequency | MP | MPx f |
|  | 4 | 45 | 180 |
|  | 8 | 55 | 440 |
|  | 5 | 65 | 325 |
|  | 3 | 75 | 225 |

1170/20 = 58.5 kg

**Skills Check 3**

1. Find the lower bound for the perimeter of this parallelogram if the measurements shown are correct to the nearest centimetre.

LB = 9.5 and 15.5

16cm

10cm

 = 50cm

1. Work out , simplifying your answer as far as possible. $\frac{19}{12}=1\frac{7}{12}$
2. Use prime factors to find the lowest common multiple of 112 and 84.

LCM = 336

1. Vicki rolls a dice 20 times. Her scores are recorded in this table. Calculate the mode, median and mean of her scores.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Score | 1 | 2 | 3 | 4 | 5 | 6 |
| Frequency | 4 | 4 | 0 | 5 | 6 | 1 |

Mode = 5

Median = 4

Mean = 3.4

1. Calculate the total surface area of a cone with radius 6cm and vertical height 8cm. Give your answer as a single multiple of π.

8cm

6cm

Curved surface = 60π

Base = 36π

Total = 96π

1. Work out , giving your answer in standard form.

 1.2 x1020

1. Work out 295.05 ÷ 7

 42.15

8. Find the nth term of the sequence: 3, 12, 27, 48…. $3n^{2}$

9. Use prime factors to find the highest common factor of 150 and 900.

 150

10. Write the recurring decimal $\dot{0.101010101….}$ as a fraction in its simplest form

$$ x=0.101010…..$$

$$ 100x=10.101010…$$

 $99x=10$

$$ x=\frac{10}{99}$$

1. **Short Exam Questions**

**Section 1 – Factorising and Simplifying**

1. Expand and simplify each of these expressions:
	1.  $15x^{2}-3x+112$
	2.  8$x^{2}-10x$
	3.  $4x+28-3x+6=x+34$
	4.  $x^{2}+9x+14$
	5.  $8x^{2}+20x-2x-5=8x^{2}+18x-5$
	6.  $25x^{2}-30x+9$
2. Factorise each of these expressions by removing common factors:
	1.  $y(3x+5)$
	2.  $6x^{2}(2x-3)$
	3.  $4y(2x+1)$
3. In an exam, Robert factorises the expression 18y3-36y2 to give the answer 9y(2y2-4y). Explain why he would **not** receive full marks. This has not FULLY been factorised. It should be $18y^{2}(y-2)$
4. Factorise each of these quadratic expressions using double brackets:
	1.  $(x+7)(x+2)$
	2.  $(x-7)(x+10)$
	3.  $(x-10)(x+9)$
	4.  $(x-6)(x-5)$
5. Factorise each of these harder quadratic expressions:
	1.  $(2x+1)(x+5)$
	2.  $(3x-7)(x-1)$
	3.  $(5x-2)(x+3)$
6. Factorise each of these using the difference of two squares:
	1.  $(y-6)(y+6)$
	2.  $(7y+9)(7y-9)$
	3.  $(x+3y)(x-3y)$
7. Factorise each of these expressions **fully**:
	1.  $5(x+2)(x+4)$
	2.  $y(y^{2}-16)$
	3.  $6y(y-3)^{2}$
8. Use factorisation to work out  **without** a calculator.

Show every step in your working.

(53+47) (53-47)=

100 x 6 = 600

1. Anna’s back garden consists of a rectangular lawn measuring 9 metres by 7 metres,

surrounded by a gravel path of width x metres.

Find, and simplify, an expression for the total area of the garden.

Path = ($7+2x)(9+2x)$

= $63+14x+18x+4x^{2}$

= $4x^{2}+32x+63$

Gravel Path

Lawn

9

7

x

x

**Section 2 - Solving Equations**

1. Solve each of these equations.
	1.  $x=4.4$
	2.  $y=15$
	3.  $m=7$
	4.  $x=2$
	5.  $k=7$
	6.  $m=-4$





1. The perimeter of the rectangle

shown opposite is 38cm.

What is the value of x.

2(3+2x)+2(4x-5) = 38

6+4x+8x-10 = 38

12x = 42

X = 3.5

1. Solve each of these equations.
	1.  d = 45
	2.  y = 1
	3.  m = 18
2. Solve the equation .

$$\frac{11x-22+4x+12}{44}=10$$

$$\frac{15x-10}{44}=10$$

$$15x=450$$

$$x=30$$

1. Simplify each of these algebraic fractions as far as possible.
	1.  $\frac{h}{3}$
	2.  $\frac{1}{4c}$
	3.  $\frac{5y-7}{4}$
	4.  $\frac{(x+5)(x+2)}{6(x+2)}= \frac{(x+5)}{6}$
	5.  $\frac{(x+4)(x-4)}{x(x+4)}= \frac{(x-4)}{x}$
2. Simplify these fractional multiplications and divisions.
	1.  $\frac{3}{4}$
	2.  $\frac{3}{2m^{2}}$
	3.  $\frac{x+1}{3}× \frac{9}{x^{2}+5x+4}=\frac{9(x+1)}{3(x+1)(x+4)}= \frac{3}{x+4}$
3. Write each of these expressions as a single algebraic fraction. Simplify your answers where appropriate.
	1.  $\frac{7y}{12}$
	2.  $\frac{6\left(x+4\right)+5(x-1)}{30}= \frac{6x+24+5x-5}{30}=\frac{11x+9}{30}$
	3.  $\frac{x+4+2x-6}{(x-3)(x+4)}=\frac{3x-2}{(x-3)(x+4)}$
4. You are given the equation .

Setting out each stage of your working clearly, show that this equation can be transformed into the form .

$$120x+480+180x-540=15(x-3)(x+4)$$

$$300x-60=15(x^{2}+x-12)$$

$20x$−4=$x^{2}+x-12$

Which gives $x^{2}-19x-8=0 as required$

**Section 3 – Inequalities and Simultaneous Equations**

 Solve these inequalities and represent the solutions on a number line.
a) 3x < 24 $x<8$

b) 2x – 5 > 17 $ x>11$

c) 2(x + 5) ≤ 16 $x\leq 3$

d) 7x – 5 ≥ 3x + 3 $x\geq 2$

e) 3x + 1 < x + 3 $x<1$

Write down inequalities to describe each of these number lines.

-3

-1

0

-2

2

1

4

3

 $x>-2$

-3

-1

0

-2

2

1

4

3

 $-2\leq x<3$

1. Write down all the integer values of x included in the inequality .

-4, -3, -2, -1, 0, 1, 2

1. Write down all the integer values of x included in the inequality .

-2, -1, 0, 1

1. Tickets to a fair cost £4.75 for adults and £2.50 for children. A coach party of 48 people arrives and their tickets cost a total of £147. Form a pair of simultaneous equations, and solve them to find the number of adults and children on this coach.

 4.75A + 2.50C =147

 A+ C = 48

 4.75A+2.5C =147

 2.50A+2.5C=120

 2.25A = 27

 Adults=12

 Children=36

1. The length of a rectangle is 6cm more than its width.

The area of the rectangle is 55cm2.

* + - * 1. Form a quadratic equation to represent this information.

$$ w\left(w+6\right)=55$$

$ w^{2}+6w $= 55

$ w^{2}+6w$-55 = 0

$ (w+11)(w$−5) = 0

$ w= $−11 or 5

$ Since length it can^{'}t be$−11. W = 5

* + - * 1. Solve your equation to find the dimensions of this rectangle.

 Width = 5, Length = 11

1. Solve these simultaneous equations algebraically:



Multiplying to get equal numbers of *y* gives:



Adding these gives  so 

Substituting into the first equation gives

 

1. Bobby buys 28 tins of beans. Large tins cost 73p and small tins cost 49p. Altogether his beans cost £17.80.

How many large tins and how many small tins did he buy?

The question leads to these simultaneous equations:



 Multiplying the first equation by 49:



 Subtracting these gives:

 

 So Bobby buys 17 large and 11 small tins of beans

1. Solve the inequality  and show your answer on a number line.



8

7

10

9

 

 

1. Solve these non-linear simultaneous equations:

 

ANS 





, 

Solutions: x = -10 with y = 49 and x = 2 with y = 13.

**Section4 - Mixed Questions**









Find and simplify expressions for the **perimeter** and **area** of

this compound shape.

Perimeter = total of all six sides =  or 

Area =  or 

The perimeter of the rectangle shown opposite is 32cm.

Find the value of *x* and the **area** of the rectangle.





 so  so  so . Length = 13cm, width = 3cm so area = 39cm2

Ben cycles from Acton to Beeswell and back again, a distance of 60 miles in each direction. On the outward journey he averages x mph but on the return journey his average is 3mph **less**. The total journey takes 12 hours. Write this information as an equation, and then show that it can be rearranged to make . (Remember that time taken = )

Question leads to . Cross-multiplying gives  which leads to . Dividing by 12 gives , as required.

Simplify as far as possible

1. 
2. 
3. 

ANS:  b. 

1.  or
2. Simplify as far as possible;
	1. =  b. = 
3. Expand and simplify the expression .

ANS===

1. Natalie is ‘**a**’ years old. Write down expressions in terms of **a** for the following people’s ages:
a) Joyce, who is 10 years older than Natalie a+10
b) John, who is half Natalie’s age. a/2
c) Gavin, who is twice *Joyce’s* age. 2(a+10)
d) Steven, who is 4 years older than *John*. $\frac{a}{2}+4$
2. The angles in a triangle are x, 3x and 5x.
Write an equation to find the value of x.

$$x+3x+5x=180$$

$$9x=180$$

$$x=20$$

Write down the size of each angle in the triangle.

20o, 60o, 100o

1. The four angles of a quadrilateral are 45°, 105°, (4*x* – 15)° and 5*x*°.
2. Form an equation, in terms of *x*, using this information.

$$45+105+4x-15+5x=360$$

$$135+9x=360$$

$$x=25$$

 b) Solve your equation and work out the size of the largest angle of the quadrilateral.

 Angles= 45, 105, 85, 100 so largest angle = 100o

1. The length of a rectangle is 5cm more than its width. The area of the rectangle is 18cm2. Form and solve a quadratic equation to find the width of this rectangle. Give your answer correct to two decimal places.





 (Watch minus signs here!)



Width = 2.42cm (or -7.42cm)

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