Write your name here

Surname JustMaths

~~Other names~~ Solutions

Centre Number

Candidate Number

Edexcel GCSE

Mathematics A

Algebraic Proof

Higher Tier

Even numbers are represented by $2n$ Odd numbers are represented by $2n - 1$

Consecutive numbers ar represented by n, $n + 1$, $n + 2$, …

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Calculators must not be used**.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.
- Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

1. Tanaka says 'When you multiply an odd number and an even number together, you will always get an odd number'.

Show that Tanaka is wrong.

odd × even = even

i.e. $3 \times 2 = 6$

$2 \times 3 = 6$

$1 \times 2 = 2$

$2 \times 1 = 2$

(Total 2 marks)

---

2. Tarish says,

'The sum of two prime numbers is always an even number'.

He is **wrong**.
Explain why.

If one of the prime numbers is "2" this is not the case i.e. $3 + 2 = 5$ which is odd. (2 is the only even prime number)

(Total 2 marks)

---

3. The n th even number is $2n$.
The next even number after $2n$ is $2n + 2$.

(a) Explain why.

2n+1 would be odd so because even numbers are 2 "apart"
the next one would be $2n \underline{+2}$

(1)

(b) Write down an expression, in terms of n, for the next even number after $2n + 2$.

$2n+2+2$

$2n+4$

(1)

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6.

let the first even number be 2n so
2nd even number is $2n+2$
3rd even number is $2n+4$

$2n + 2n+2+2n+4 = 6n+6 = 6(n+1)$

Factor of 6

(3)

(Total 5 marks)

4. Here are the first 4 lines of a number pattern.

$1 + 2 + 3 + 4$	$=$	$(4 \times 3) - (2 \times 1)$
$2 + 3 + 4 + 5$	$=$	$(5 \times 4) - (3 \times 2)$
$3 + 4 + 5 + 6$	$=$	$(6 \times 5) - (4 \times 3)$
$4 + 5 + 6 + 7$	$=$	$(7 \times 6) - (5 \times 4)$

n is the first number in the nth line of the number pattern.

Show that the above number pattern is true for the four consecutive integers n, $(n + 1)$, $(n + 2)$ and $(n + 3)$.

$\boxed{\text{LHS}}$

$n + (n+1) + (n+2) + (n+3)$

$= 4n+6$

$\boxed{\text{RHS.}}$

$(n+2)(n+3) - n(n+1)$

$= \cancel{n^2} + 3n + 2n + 6 \quad - (\cancel{n^2} + n)$

$= 5n + 6 - n$

$= 4n+6$

$\therefore$ LHS = RHS.

(Total 4 marks)

5. n is a whole number.

Prove that $n^2 + (n + 1)^2$ is always an odd number.

$$n^2 + (n+1)^2 = n^2 + (n+1)(n+1)$$
$$= n^2 + n^2 + 2n + 1$$
$$= 2n^2 + 2n + 1$$

n^2 is always even

$2n$ is always even

$2n^2$ is always even

Even + Even + Odd
= Odd

(Total 2 marks)

---

6. n and a are integers.

Explain why $(n^2 - a^2) - (n - a)^2$ is always an integer.

$$(n-a)(n-a) = n^2 - na - na + a^2$$

$$n^2 - a^2 - (n^2 - 2na + a^2)$$

$$\cancel{n^2} - a^2 - \cancel{n^2} + 2na - a^2$$

$$2na - 2a^2 = 2a(n-a)$$

if a and n are integers
$2a(n-a)$ will also be an integer

(Total 2 marks)

---

7. n is an integer greater than 1.

Use algebra to show that $(n^2 - 1) + (n - 1)^2$ is always equal to an even number.

$(n-1)(n-1)$

$n^2 - 2n + 1$

$n^2 - 1 + n^2 - 2n + 1$

$2n^2 - 2n$

$2n(n-1)$

given n is greater than 1 $2n(n-1)$ is always even.

(Total 4 marks)

---

8. Prove that the difference between the squares of any two consecutive even numbers is always an odd number multiplied by 4.

$(2n+2)(2n+2)$

$4n^2 + 4n + 4n + 4$

n = any number

$2n$ = any even number

next even number = $2n+2$

$(2n)^2 + (2n+2)^2 = 4n^2 + 4n^2 + 8n + 4$

$= 8n^2 + 8n + 4$

$= 4(2n^2 + 2n + 1)$

EVEN + EVEN + 1

$4 \times$ ODD

(Total 4 marks)

---

9. Prove algebraically that the sum of the squares of two consecutive integers is always an odd number.

Let n be any integer

$n^2 + (n+1)^2$

$n^2 + n^2 + 2n + 1$

$2n^2 + 2n + 1$

EVEN + EVEN + 1

ALWAYS ODD

(Total 3 marks)

***10.** Prove that the sum of the squares of any two odd numbers is always even.

$2n+1$

$2n+3$

$(2n+1)^2 + (2n+3)^2$

$= 4n^2 + 2n + 2n + 1 + 4n^2 + 6n + 6n + 9$

$= 8n^2 + 16n + 10$

$2(4n^2 + 8n + 5)$

always EVEN

(Total 4 marks)

---

11. Show that $(n + 3)^2 - (n - 3)^2$ is an even number for all positive integer values of n.

$(n+3)^2 = n^2 + 6n + 9$

$(n-3)^2 = n^2 - 6n + 9$

$n^2 + 6n + 9 - n^2 + 6n - 9$

$= 12n.$

$= 2(6n)$

↑

always EVEN

(Total 3 marks)

***12.** Prove that

$$(7n + 3)^2 - (7n - 3)^2$$

is a multiple of 12, for all positive integer values of n.

$(7n+3)^2 = 49n^2 + 21n + 21n + 9$

$(7n-3)^2 = 49n^2 - 21n - 21n + 9$

$49n^2 + 42n + 9 - 49n^2 + 42n - 9$

$= 84n$

$= 2(42n)$

alwayseven

(Total 3 marks)

$2n+1 \quad 2m+1$

***13.** Prove algebraically that the product of two odd numbers is **always** an odd number.

$\times$

Let $2n+1$ be any odd number
and $2m+1$ be another odd number

$$(2n+1)(2m+1) = 4nm + 2n + 2m + 1$$
$$= 2(2mn + n + m) + 1$$

means $2(2mn+n+m)$ will always be even

Even + 1 = odd

(Total 3 marks)

---

14. Prove algebraically that the sum of any two odd numbers is even.

Let $2n+1$ be any odd number
$2m+1$ is another odd number

$$(2n+1) + (2m+1) = 2n + 1 + 2m + 1$$
$$= 2n + 2m + 2$$
$$= 2(n+m+1)$$

means the result will always be even.

(Total 3 marks)

***15.** Prove algebraically that

$-2n-1$

$$(2n+1)^2 - (2n+1) \quad \text{is an even number}$$

for all positive integer values of n.

$(2n+1)^2 = (2n+1)(2n+1)$
$= 4n^2 + 2n + 2n + 1$
$= 4n^2 + 4n + 1$

$4n^2 + 4n + 1 - 2n - 1$
$= 4n^2 + 2n$
$= 2n(n+1)$

x2 means the result will always be an even number

(Total 3 marks)

***16.** Given that a and b are two consecutive even numbers, prove algebraically that

$$\left(\frac{a+b}{2}\right)^2 \text{ is always 1 less than } \frac{a^2+b^2}{2}.$$

Let 2n be any even numbe
next even numbe would be 2n + 2

$$\left(\frac{a+b}{2}\right)^2 \Rightarrow \left(\frac{2n+2n+2}{2}\right)^2 = \left(\frac{4n+2}{2}\right)^2 = (2n+1)^2$$
$$= (2n+1)(2n+1)$$
$$= 4n^2 + 4n + 1$$

$$\frac{a^2+b^2}{2} = \frac{(2n)^2 + (2n+2)^2}{2} = \frac{4n^2 + 4n^2 + 8n + 4}{2}$$
$$= \frac{8n^2 + 8n + 4}{2} = 4n^2 + 4n + 2$$

$\therefore$ $4n^2 + 4n + 1$ is 1 less than $4n^2 + 4n + 2$
✓ ✓ ✓ ✓

(Total 5 marks)

17. Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of n.

$(3n+1)^2 - (3n-1)^2$

$(3n+1)(3n+1) - (3n-1)(3n-1)$

$(9n^2+3n+3n+1) - (9n^2-3n-3n+1)$

$\cancel{9n^2} + 6n + 1 - \cancel{9n^2} + 6n - 1$

$= 12n = 4(3n)$

always a multiple of 4

(Total 3 marks)

***18.** Prove that the sum of the squares of two consecutive odd numbers is never a multiple of 8.

any even number is $2n$ so any odd number is $2n-1$ and the next odd number is $2n+1$

$(2n-1)^2 + (2n+1)^2 = (2n-1)(2n-1) + (2n+1)(2n+1)$

$= 4n^2 - 2n - 2n + 1 + 4n^2 + 2n + 2n + 1$

$= 8n^2 + 2$

$= 2(4n^2 + 1)$

always a multiple of 2 not 8

(Total 4 marks)

---

19. Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

$(2n+3)^2 - (2n-3)^2$

$(2n+3)(2n+3) - (2n-3)(2n-3)$

$(4n^2+6n+6n+9) - (4n^2-6n-6n+9)$

$\cancel{4n^2}+12n\cancel{+9} - \cancel{4n^2}+12n\cancel{-9}$

$24n = 8(3n)$

$\uparrow$

multiple of 8

(Total 3 marks)

---

20. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

let any number be n
the next/consecutive number is $n+1$

$n + n + 1 = 2n + 1$

$2n$ is always even, so an even number $+1$ will always be odd

(Total 3 marks)

---

21. Umar thinks $(a+1)^2 = a^2 + 1$ for all values of a.

(a) Show that Umar is wrong.

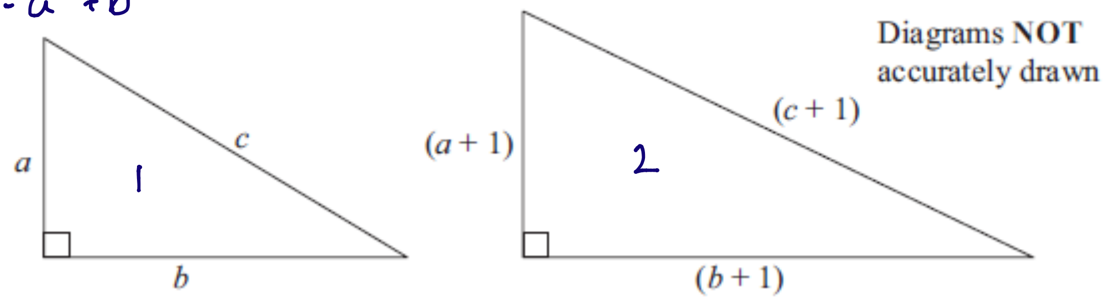
$(a+1)^2 = (a+1)(a+1)$

$= a^2 + 2a + 1$ which is different to $a^2 + 1$

(2)

Here are two right-angled triangles.
All the measurements are in centimetres.

$c^2 = a^2 + b^2$



(b) Show that $2a + 2b + 1 = 2c$

from triangle 2 $(c+1)^2 = (a+1)^2 + (b+1)^2$

$c^2 + 2c + 1 = a^2 + 2a + 1 + b^2 + 2b + 1$

substitute $c^2 = a^2 + b^2$

$\cancel{a^2} + \cancel{b^2} + 2c \underset{-1}{+1} = \cancel{a^2} + 2a + \cancel{b^2} + 2b \underset{-1}{+2}$

$2c = 2a + 2b + 1$

As required.

(3)

a, b and c cannot all be integers.

(c) Explain why.

$2c = 2a + 2b + 1$

always even   always even

LHS is always even but the RHS is an even number + 1 which will be odd

(1)

(Total 6 marks)

***22.** Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

let n be anyinteger
n+1 is the next integer
Difference of squares
$(n+1)^2 - n^2$
$n^2 + 2n + 1 - n^2 = 2n + 1$ $\longleftrightarrow$ equal.

Sum of integers
$n + n + 1$
$= 2n + 1$

(Total 4 marks)

TOTAL FOR PAPER IS 71 MARKS

BLANK PAGE