Mathematics
2019 Practice Paper
Paper 2 (Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Calculators may be used**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

1 (a) Given $\dfrac{x^6}{x^a} = x^8$

Find the value of a.

$a = \underline{\quad -2 \quad}$

(1)

(b) Simplify $\dfrac{5b^3 \times 2b^6}{3b^4}$

$$\dfrac{10b^9}{3b^4} = \dfrac{10b^5}{3}$$

$\underline{\dfrac{10b^5}{3}}$

(2)

(c) Simplify $(2m^2)^4$

$\underline{16m^8}$

(2)

(Total for question 1 is 5 marks)

2 **(a)** Write 9870000 in standard form.

$$9.87 \times 10^{6}$$

(1)

(b) Work out the value of $(9.2 \times 10^6) \div (3.4 \times 10^8)$
Give your answer in standard form to 3 significant figures.

$$0.0271 \quad (3sf)$$

$$2.71 \times 10^{-2}$$

(2)

(Total for question 2 is 3 marks)

3 Charlie invests £5600 for 4 years in a savings account.
She gets 2% per annum compound interest.

How much money does Charlie have at the end of 4 years.

$$5600 \times 1.02^{4}$$

$$= £6061.62$$

£ 6061.62

(Total for question 3 is 2 marks)

4 A football team sell home shirts and away shirts.
The ratio of home shirts to away shirts sold is 5:1   *6 parts*

The shirts can either be adult's shirts of children's shirts.
The ratio of adults shirts sold to children's shirts sold is 3:2   *5 parts*

What proportion of shirts sold are children's home shirts?

HOME SHIRTS $\dfrac{5}{6}$

CHILDRENS SHIRTS $\dfrac{2}{5}$

$$\dfrac{5}{6} \times \dfrac{2}{5} = \dfrac{1}{3}$$

$\dfrac{1}{3}$
....

(Total for question 4 is 2 marks)

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5 The average house price in London in 2017 was £474902
The average house price in London in 2018 was £469538

Calculate the percentage change in house prices between 2017 and 2018.

$$\dfrac{change}{original} \times 100$$

$$\dfrac{469538 - 474902}{474902} \times 100 = -1.1294...$$

$$= -1.13\%$$
$$(2dp)$$

– 1.13
....%

(Total for question 5 is 2 marks)

6 In London potatoes cost £0.45 per lb.
In Dublin potatoes cost €1.48 per kilogram.

$$1 \text{ kg} = 2.2 \text{ lbs}$$
$$£1 = €1.15$$

In which city are potatoes better value for money, London or Dublin?
You must show your working.

DUBLIN

€ $1.48 \div 2.2 = €0.67\dot{2}$ per lb

$0.67\dot{2} \div 1.15 = £0.58$ per lb

LONDON

£0.45 per lb

LONDON

(Total for question 6 is 3 marks)

7 Andy and Bruce share some sweets in the ratio 9:4. 13 PARTS
Andy gets *A* sweets
Bruce gets *B* sweets

Carla and David share the same amount of sweets as Andy and Bruce.
They share their sweets in the ratio 5:2. 7 PARTS

Carla gets *C* sweets
David gets *D* sweets

Find *A:B:C:D*

A : B
(9 : 4) ×7

63 : 28

C : D
(5 : 2) × 13

65 : 26

63 : 28 : 65 : 26
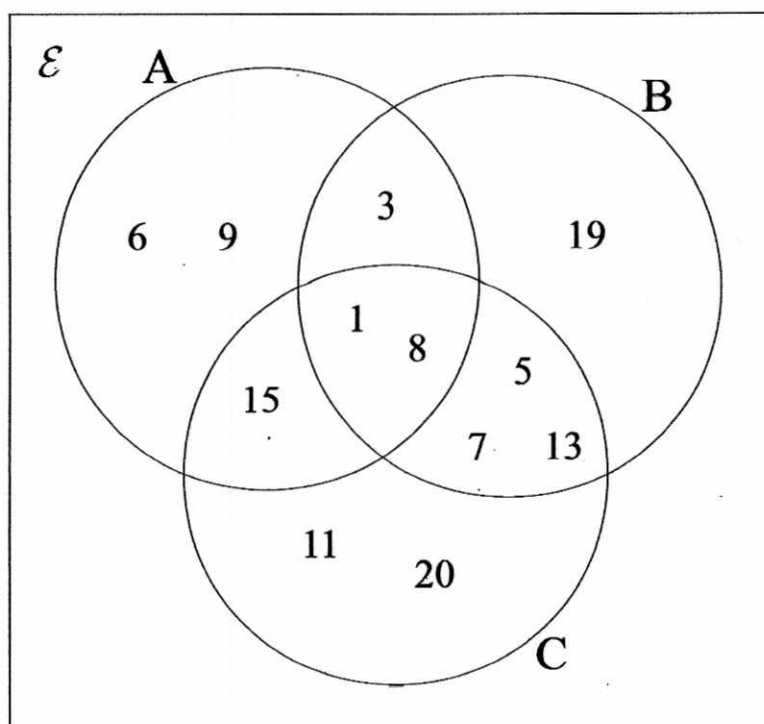
(Total for question 7 is 3 marks)

8 Here is a Venn diagram.

(a) List the members of A ∩ B

..........1, 3, 8..........

A number is chosen at random from $\mathcal{E}$.

(b) Find $P(B \cup C)$

$\frac{10}{12}$ OR $\frac{5}{6}$

(Total for question 8 is 3 marks)

9 Adam is measuring the heights in cm of his tomato plants.

Height (cm)	m	Frequency (f)	midpoint × f
140 < h ⩽ 150 145		7	1015
150 < h ⩽ 160 155		10	1550
160 < h ⩽ 170 165		15	2475
170 < h ⩽ 180 175		19	3325
180 < h ⩽ 200 190		9	1710
		60	10075

Estimate the mean height.

$$\frac{10075}{60} = \frac{2015}{12} = 167.91\dot{6}$$

167.9 cm (1dp)

(Total for question 9 is 3 marks)

10 100ml of liquid A and 200ml of liquid B are mixed together to make liquid C.

Liquid A has a density of 0.8g/ml. D

Liquid B has a density of 1.1 g/ml. D

Work the density of liquid C.

$$density = \frac{mass}{volume}$$

Liquid A: mass = density × volume

$$= 0.8 \times 100$$

$$= 80g$$

Liquid B   mass = 1.1 × $\frac{200}{\cancel{220}}$ = 220g

Density = $\frac{Total\ Mass}{Total\ Volume}$

$$= \frac{80 + 220}{100 + 200} = 1$$

....................1.................g/ml

(Total for question 10 is 4 marks)

11 Change 45000cm³ into m³.

Scale factor length × 100

Scale factor volume × 100^3

$$45000 \div 100^3 = 0.045$$

$$0.045 \ m^3$$

(Total for question 11 is 2 marks)

12
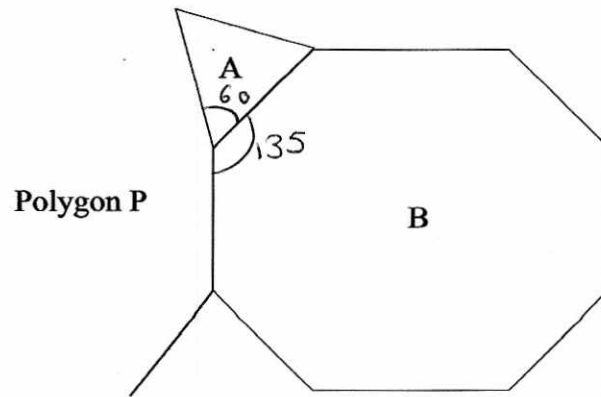
Shape A is a regular triangle. Shape B is a regular octagon.

Another regular polygon, P, is shown on the diagram.

How many sides does polygon P have?

You must show your working.

Each angle in equilateral triangle = 60°

Exterior angle of octagon = $\frac{360}{8}$ = 45°

Interior angle of octagon = 180 − 45 = 135°

Interior angle of P = 360 − 60 − 135 = 165°

Exterior angle of P = 180 − 165 = 15°

$\frac{360}{15}$ = 24 sides

.........24.........

(Total for question 12 is 4 marks)

13   On the grid shade the region that satisfies all these inequalities

$$y \geq x - 1 \qquad\qquad x \leq 6 - 2y \qquad\qquad x \geq -3$$

Label the region **R**.

$y \geqslant x - 1$

x	0	2	4
y	-1	1	3

$x \leqslant 6 - 2y$

y	0	2	4
x	6	2	-2
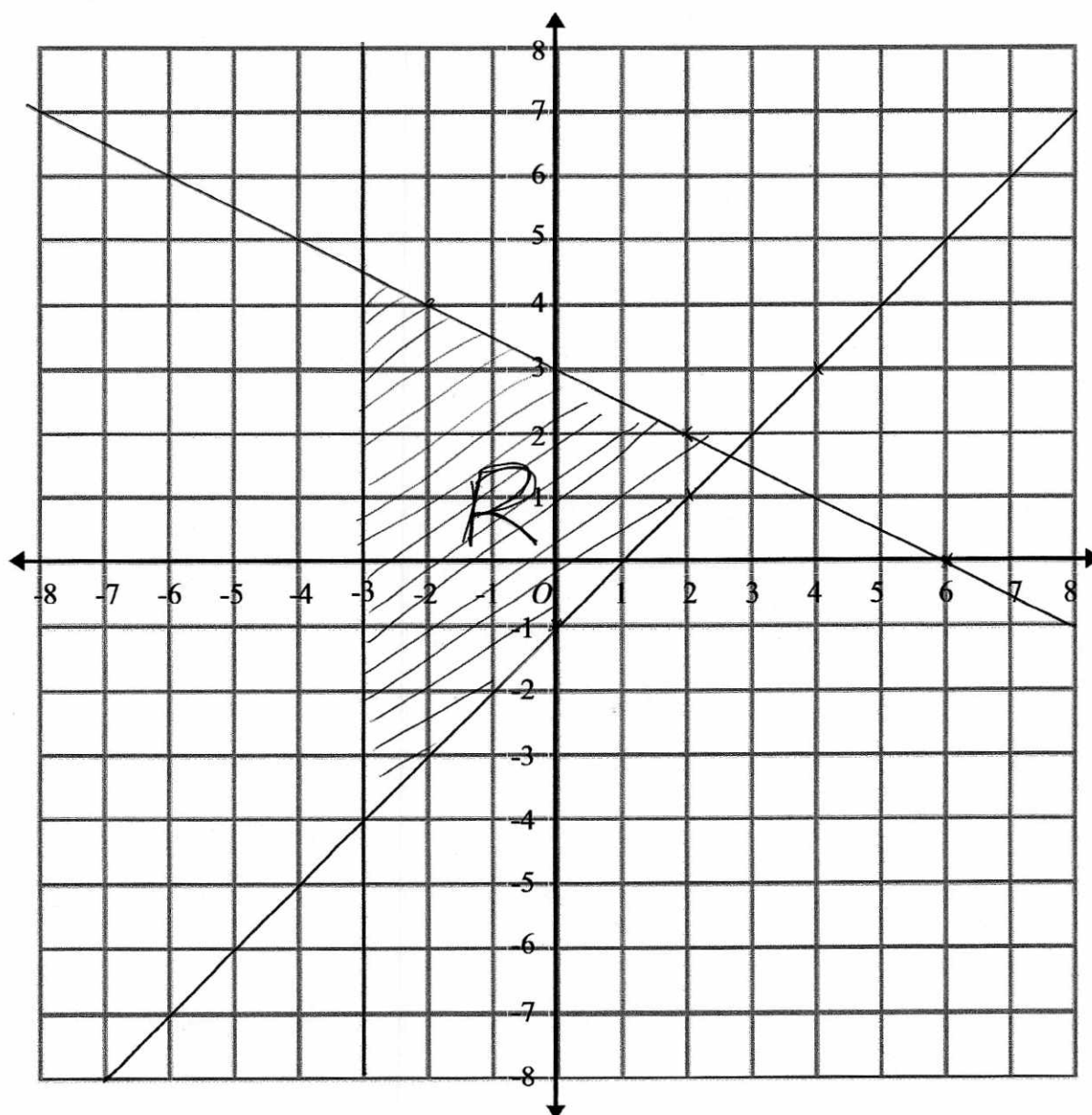
(Total for question 13 is 4 marks)

14 Prove algebraically that the recurring decimal $0.5\overset{\cdot\cdot}{2}\overset{\cdot}{7}$ can be written as $\dfrac{29}{55}$

$$0.5\overset{\cdot}{2}\overset{\cdot}{7} = x$$

$$5.\overset{\cdot}{2}\overset{\cdot}{7} = 10x$$

$$527.\overset{\cdot}{2}\overset{\cdot}{7} = 1000x$$

$$522 = 990x$$

$$x = \frac{522}{990} = \frac{29}{55}$$

(Total for question 11 is 2 marks)

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15 Using $\quad x_{n+1} = \sqrt[3]{9 + 8x_n}$

With $x_0 = 3$

Find the values of x_1, x_2 and x_3.

$$x_1 = \sqrt[3]{9 + 8(3)} = 3.20753433$$

$$x_2 = \sqrt[3]{9 + 8(Ans)} = 3.26048413$$

$$x_3 = \sqrt[3]{9 + 8(Ans)} = 3.273668236$$

$x_1 = \underline{3.20753433}$

$x_2 = \underline{3.26048413}$

$x_3 = \underline{3.273668236}$

(Total for question 15 is 3 marks)

16 The flight from London to Dubai takes 7 hours, to the nearest ten minutes.
The distance from London to Dubai is 3400 miles to the nearest 100 miles.

John says the average speed is definitely less than 500 miles per hour.

Is John correct?

You must show your working.

Time

6 hours 50 mins   6 hrs 55 mins   7 hours   7 hrs 5 mins   7 hours 10 mins

Distance

3300   3350   3400   3450   3500
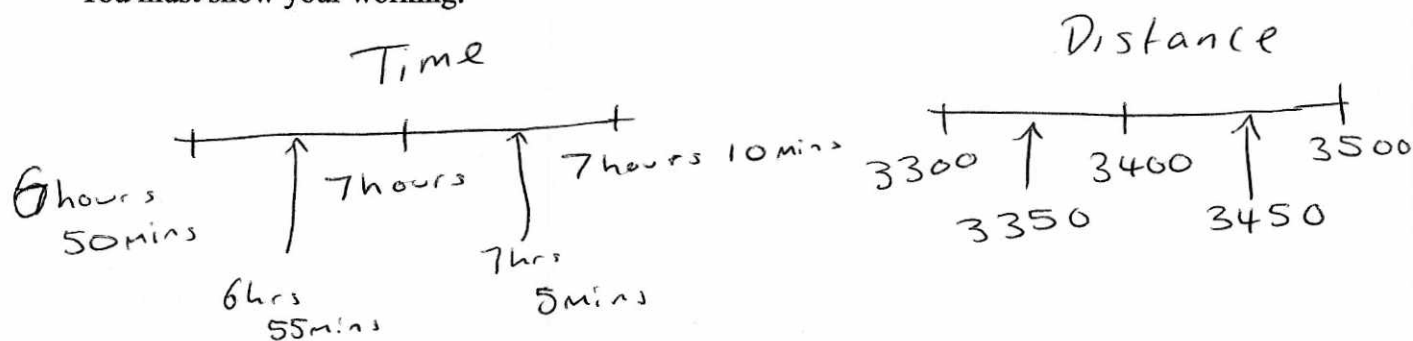
$$Upper\ Speed = \frac{Upper\ Distance}{Lower\ Time}$$

6 hrs 55
$$= 6 + \frac{55}{60} = \frac{83}{12} hrs$$
$$= 6.91\dot{6}$$

$$= \frac{3450}{6.91\dot{6}}$$

$$= 498.795 \ldots\ldots M/hr$$

John is correct Max upper bound is
less than 500 miles per hour.

(Total for question 16 is 4 marks)

17 Here is a speed-time graph.
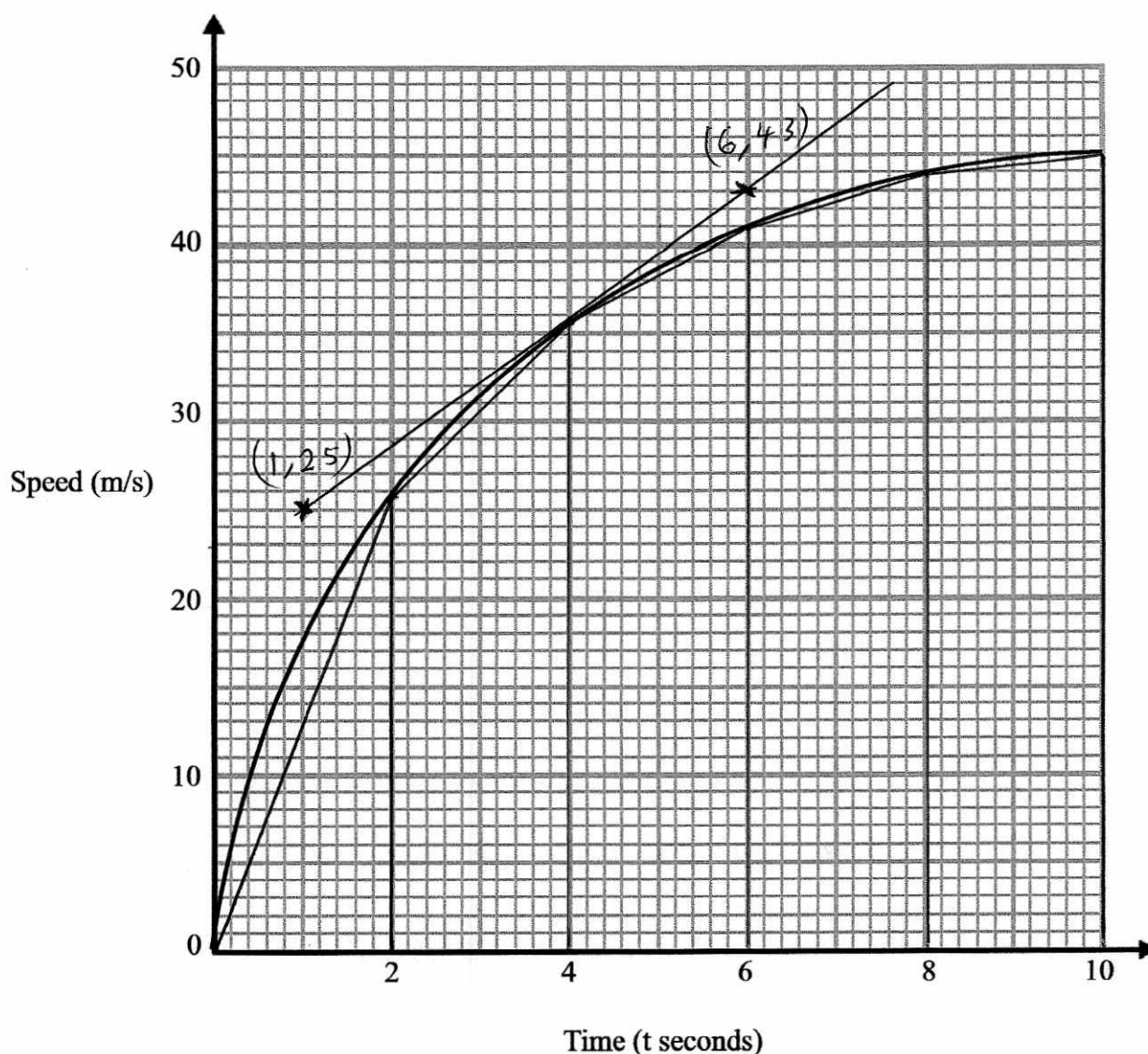
(a) Work out an estimate for the acceleration when t = 4.

$$\frac{43 - 25}{6 - 1} = 3.6 \ ms^{-2}$$

.........3.6......... ms⁻²

[3.2 - 3.8] (2)

(b) Use 5 strips of equal width to find an estimate for the distance travelled in 10 seconds.

$\frac{1}{2} \times 2 \times 25.5 = 25.5$ $\frac{1}{2}(44 + 45) \times 2 = 89$
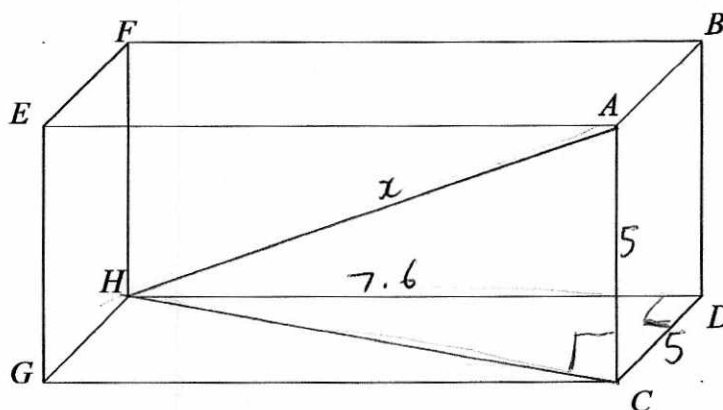
$\frac{1}{2}(25.5 + 35.5) \times 2 = 61$

$\frac{1}{2}(35.5 + 41) \times 2 = 76.5$ ....337......... m

$\frac{1}{2}(41 + 44) \times 2 = 85$ [333 - 341] (3)

(Total for question 17 is 5 marks)

18

The diagram shows a cuboid *ABCDEFGH*   All sides of square

ABCD is a square with area 25cm².   ~~AB~~ = 5 cm
The volume of the cuboid is 190 cm³.

Find the length of the diagonal AH.
Give your answer to 3 significant figures

$$\frac{190}{25} = 7.6 \text{ cm} = DH$$

3d pythagoras

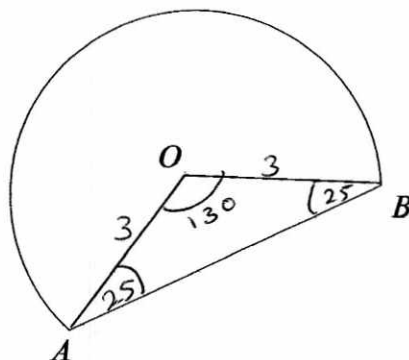$$AH^2 = 5^2 + 7.6^2 + 5^2$$

$$AH = \sqrt{5^2 + 7.6^2 + 5^2}$$

$$= 10.4 \text{ cm} \quad (3sf)$$

...10.4...cm

(Total for question 18 is 3 marks)

19

$$360 - 130$$
$$= 230°$$

The diagram shows a badge which is formed of a sector of a circle, centre O, and a triangle AOB.
OA = 3 cm.

Angle OAB = 25°

Find the total area of the badge

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(3)(3)\sin(130)$$

$$= 3.447199994 \text{ cm}^2$$

$$\text{Area of sector} = \frac{230}{360} \times \pi(3)^2$$

$$= 18.06415776 \text{ cm}^2$$

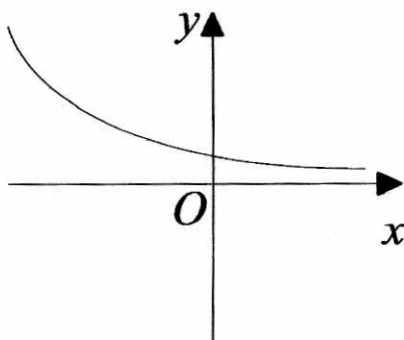$$\text{Total area} = 21.51135775 \text{ cm}^2$$

_21.5_ cm²

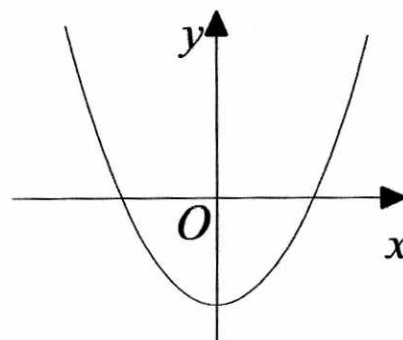**(Total for question 19 is 4 marks)**

20 Here are nine graphs.

Graph A
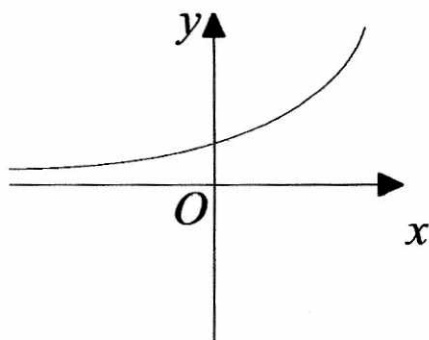
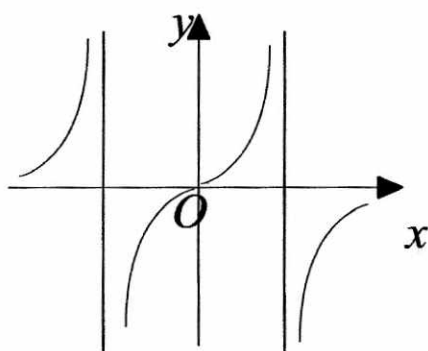**Graph B**
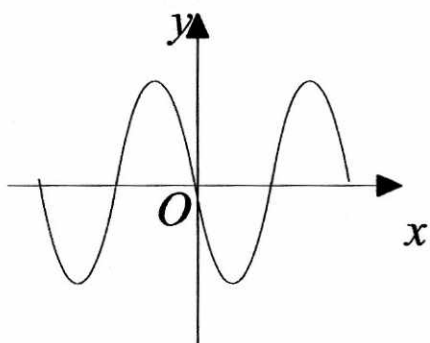
Graph C

Graph D

Graph E
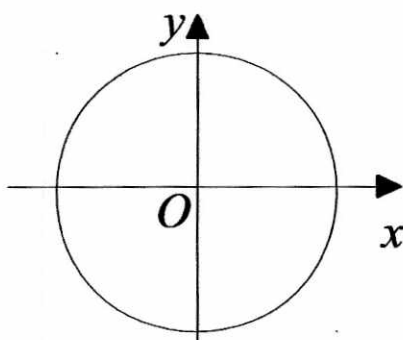
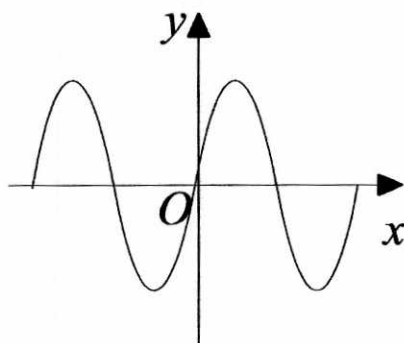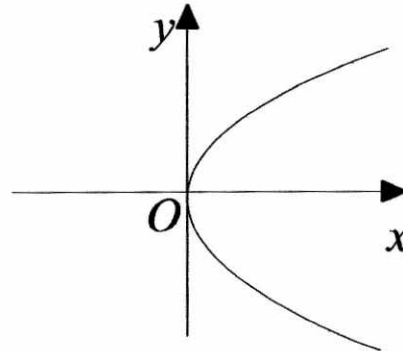**Graph F**
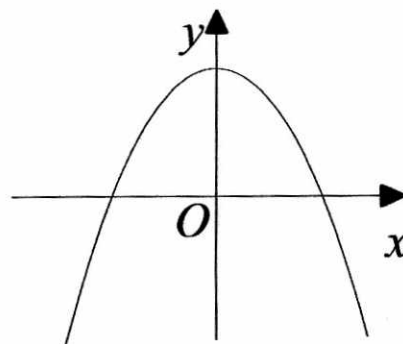
Graph G

Graph H

Graph I

Complete the table with the letter of the graph that could represent each given equation.

Equation	Graph
$y = 2\sin x$	H
$x^2 + y^2 = 10$	E
$y = 2^x$	A
$y = \tan x$	D

21 Here are the first 5 terms of a quadratic sequence.

$$4 \qquad 8 \qquad 15 \qquad 25 \qquad 38$$

Find an expression, in terms of n, for the nth term of this sequence.

$a + b + c \nearrow$ 4   8   15   25 $an^2 + bn + c$

$3a + b \nearrow$ 4   7   10

$2a \nearrow$ 3   3

$2a = 3$ $\qquad\qquad$ $3a + b = 4$ $\qquad\qquad$ $a + b + c = 4$

$a = 1.5$ $\qquad\qquad$ $3(1.5) + b = 4$ $\qquad\qquad$ $1.5 - 0.5 + c = 4$

$\qquad\qquad\qquad$ $4.5 + b = 4$ $\qquad\qquad\qquad$ $c = 3$

$\qquad\qquad\qquad\qquad$ $b = -0.5$

$$1.5n^2 - 0.5n + 3$$

(Total for question 21 is 3 marks)

$F.d = \dfrac{freq}{width}$

22   The table shows information about the speed, in mph, of 120 cars.

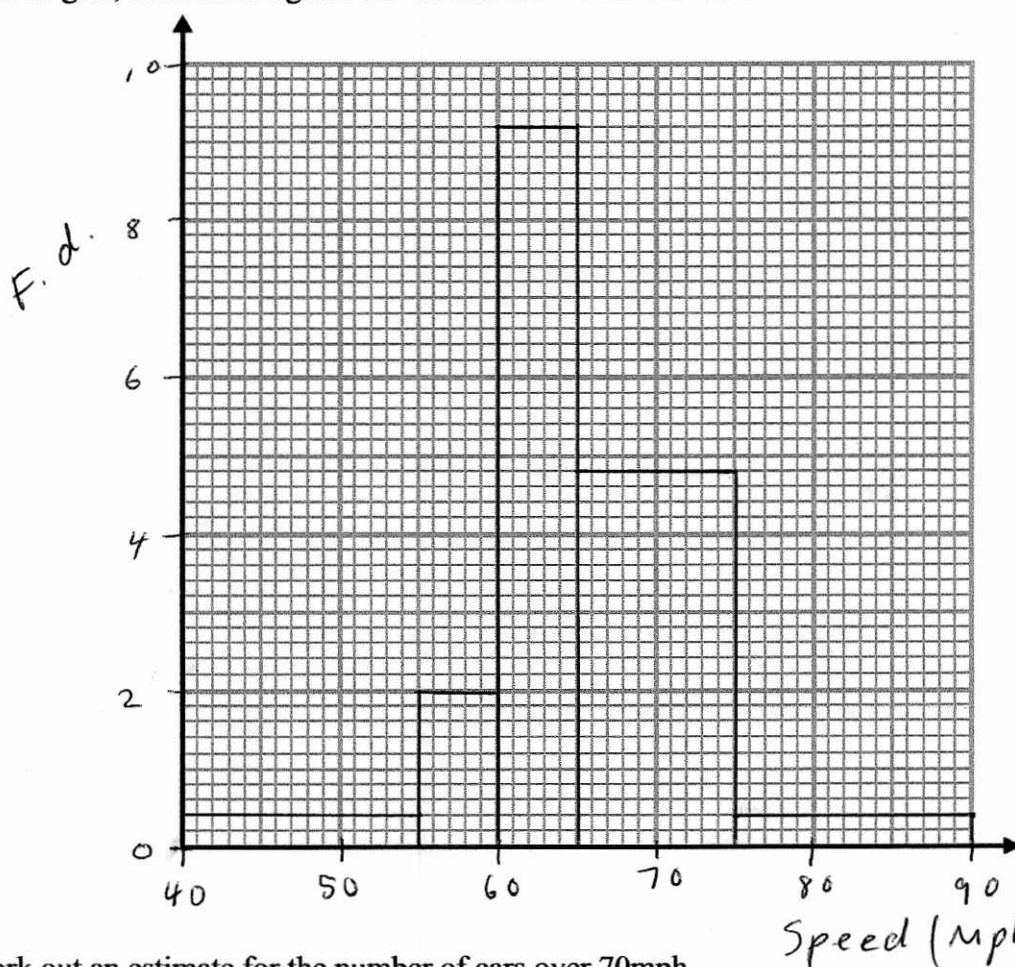| Speed (mph)	Frequency
$40 < s \leqslant 55$	6
$55 < s \leqslant 60$	10
$60 < s \leqslant 65$	46
$65 < s \leqslant 75$	48
$75 < s \leqslant 90$	6

F. d.

0.4

2

9.2

4.8

0.4

(a) On the grid, draw a histogram for the information in the table.

(3)

Speed (mph)

(b) Work out an estimate for the number of cars over 70mph.

$24 + 6$

.... 30 ....

(1)

(Total for question 22 is 4 marks)

23
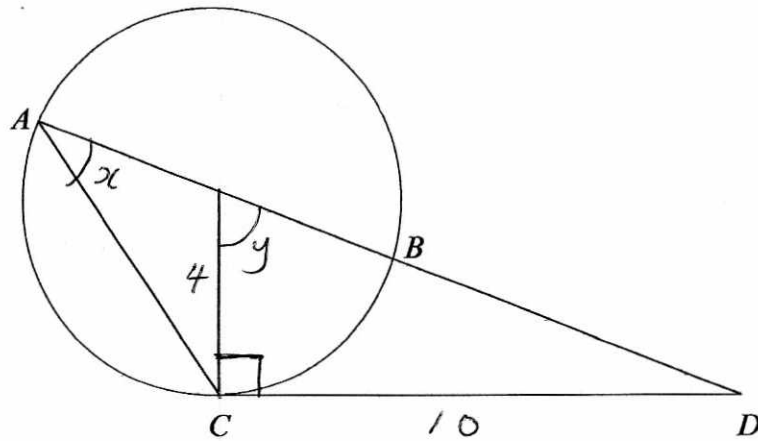
A, B and C are points on the circumference of a circle with radius 4 cm.
AB is a diameter
CD is a tangent to the circle at C
CD = 10 cm.

Find the size of angle CAB
Give your answer to 3 significant figures.

$$\tan y = \frac{10}{4}$$

$$y = \tan^{-1}\left(\frac{10}{4}\right)$$

$$= 68.19859051$$

$$x = \frac{68.19859051}{2}$$

$$= 34.1° \quad 3sf$$

34.1 °

24 Solve algebraically the simultaneous equations

$$y = 2x^2 - 3x - 7$$

$$x - 2y = 5$$

Give yours answers to 2 decimal places.

$$x = 2y + 5$$

$$y = 2(2y+5)^2 - 3(2y+5) - 7$$

$$y = 2(2y+5)(2y+5) - 6y - 15 - 7$$

$$y = 2(4y^2 + 10y + 10y + 25) - 6y - 22$$

$$y = 2(4y^2 + 20y + 25) - 6y - 22$$

$$y = 8y^2 + 40y + 50 - 6y - 22$$

$$-y \qquad\qquad\qquad -y$$

$$0 = 8y^2 + 33y + 28$$

$$y = \frac{-(33) \pm \sqrt{(33)^2 - 4(8)(28)}}{2(8)}$$

$$= \underline{-1.19} \quad , \quad \underline{-2.93}$$

$$x = 2(-1.19) + 5 \qquad\qquad x = 2(-2.93) + 5$$

$$= \underline{2.61} \qquad\qquad\qquad x = \underline{-0.86}$$

$$\underline{x = 2.61 \quad y = -1.19 \quad \text{and} \quad x = -0.86 \quad y = -2.93}$$

(Total for question 24 is 5 marks)