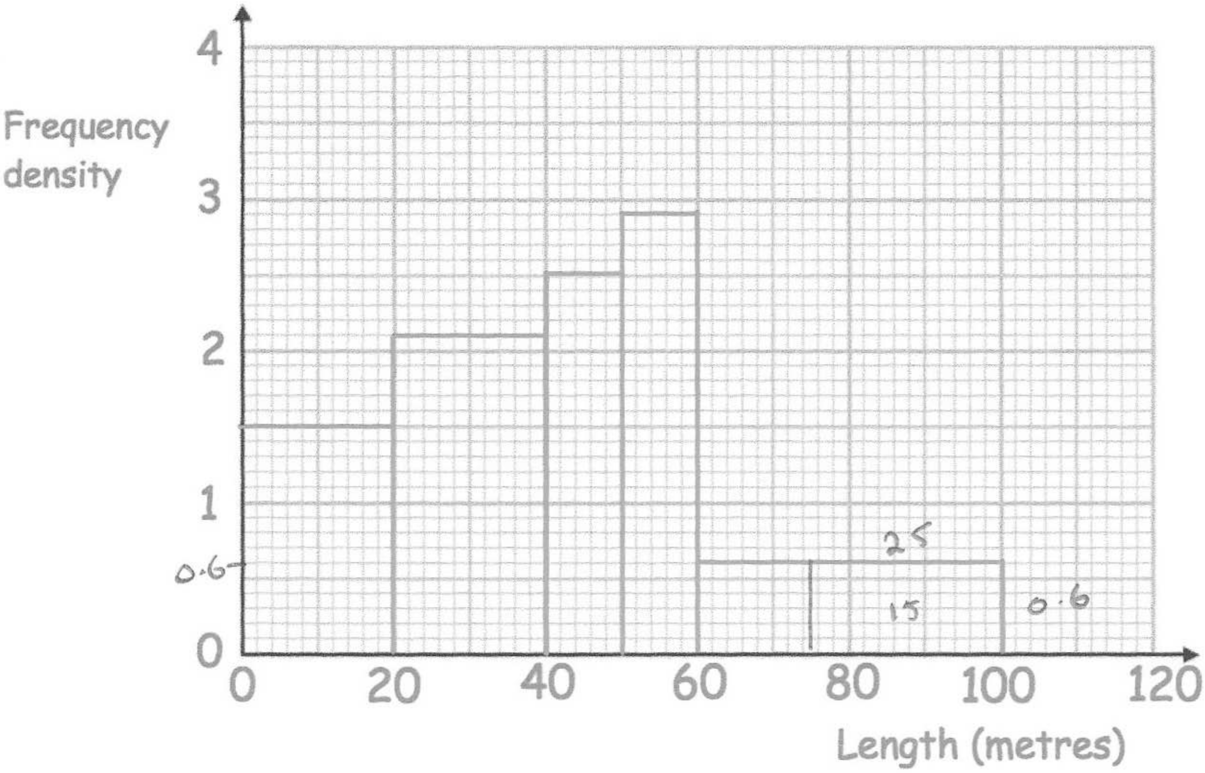
52. The histogram shows information about how far 150 children swam, when trying to get their swimming certificates.



(a) Complete this frequency table.

Length, l metres	Frequency
$0 < l \leq 20$	30
$20 < l \leq 40$	42
$40 < l \leq 50$	25
$50 < l \leq 60$	29
$60 < l \leq 100$	24

20×2.1

10×2.9

(2)

(b) 10% of the swimmers swam further than y metres.
Calculate an estimate of y.

$0.6 \times \overset{25}{\boxed{}} = 15$

10% of 150 = 15

75m

(2)

53. Two solid clay models of the Statue of Liberty are mathematically similar.

The smaller model has a height of 15cm.
The larger model has a height of 20cm.

The smaller model has a mass of 108g.

Work out the mass of the larger model.

$15\text{cm} \xrightarrow{\times 4/3} 20\text{cm}$

$108\text{g} \xrightarrow{\times (4/3)^3} 256\text{g}$

..........256..........g
(3)

54.

$$w = aT$$

Given a = 15 correct to 2 significant figures
and w = 700 correct to 2 significant figures
Calculate the upper bound for T

	LB	UB
a	14·5,	15·5
w	650,	750

$$T = \frac{W}{a}$$

$$T_{max} = \frac{W_{max}}{a_{min}}$$

$$= \frac{750}{14 \cdot 5}$$

$$T_{max} = 51 \cdot 72$$

.....T = 51·72.....
(3)

55. Factorise fully $32y^3 + 24y^2$

$8y^2(4y+3)$

$8y^2(4y+3)$

(2)

56. (a) Factorise $x^2 - x - 72$

$(x+8)(x-9)$

(2)

(b) Factorise $4x^2 + 12x - 7$

$(2x-1)(2x+7)$

(2)

57. Solve $$\frac{x+3}{4} = \frac{3}{x-1}$$

$(x+3)(x-1) = 12$

$x^2 + 3x - x - 3 = 12$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3)$

(3)

Answer $\boxed{x = -5 \text{ or } x = 3}$

58. Solve $x^2 - 6x - 20 = 0$

Give your answers to 1 decimal place.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-20)}}{2(1)}$$

$$= 3 \pm \sqrt{29}$$

$$= 8.38 \text{ or } -2.38$$

8.4 or -2.4

(3)

---

59. Here are the *n*th terms of 4 sequences.

Sequence 1	nth term	3n + 1	4 7 10
Sequence 2	nth term	5n + 10	15 20 25
Sequence 3	nth term	10n	10 20 30
Sequence 4	nth term	5n - 1	4 9 14

For each sequence state whether the numbers in the sequence are

A Always multiples of 5
S Sometimes multiples of 5
N Never multiples of 5

Sequence 1 ....S....

Sequence 2 ....A....

Sequence 3 ....A....

Sequence 4 ....N....

(4)

60. Here are the first 5 terms of a quadratic sequence

4 10 18 28 40

Find an expression, in terms of n, for the nth term of this quadratic sequence.

$a+b+c$ 4 10 18 28 40

$3a+b$ 6 8 10 12

$2a$ 2 2 2

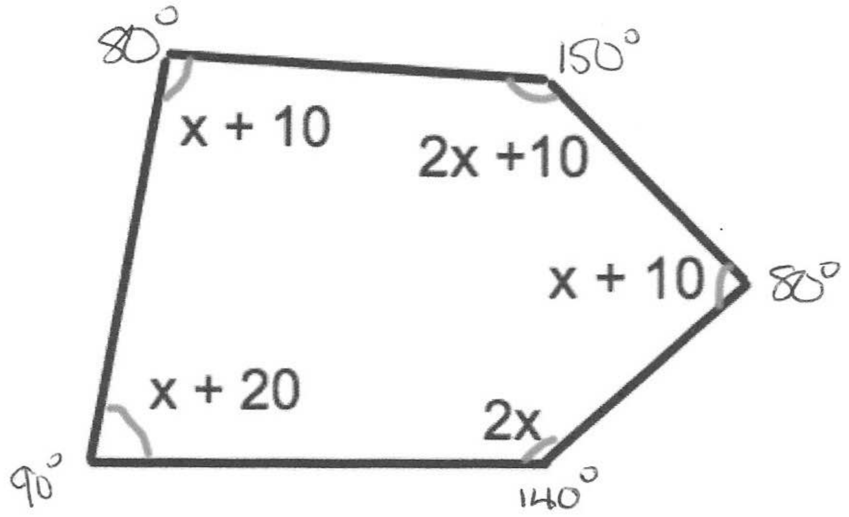
$a=1$
$b=3$
$c=0$

n^2+3n

(3)

---

61. Shown is a pentagon, with the size of each angle shown.



Find the size of the largest angle.

$7x+50=540$

$x=70°$

150 °

(4)

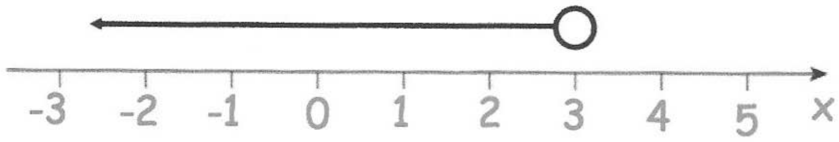
62. (a) Solve the inequality $4x + 6 \geq 2$

$4x \geqslant -4$

$x \geqslant -1$

$x \geqslant -1$
(2)

(b) Write down the inequality shown by the diagram.

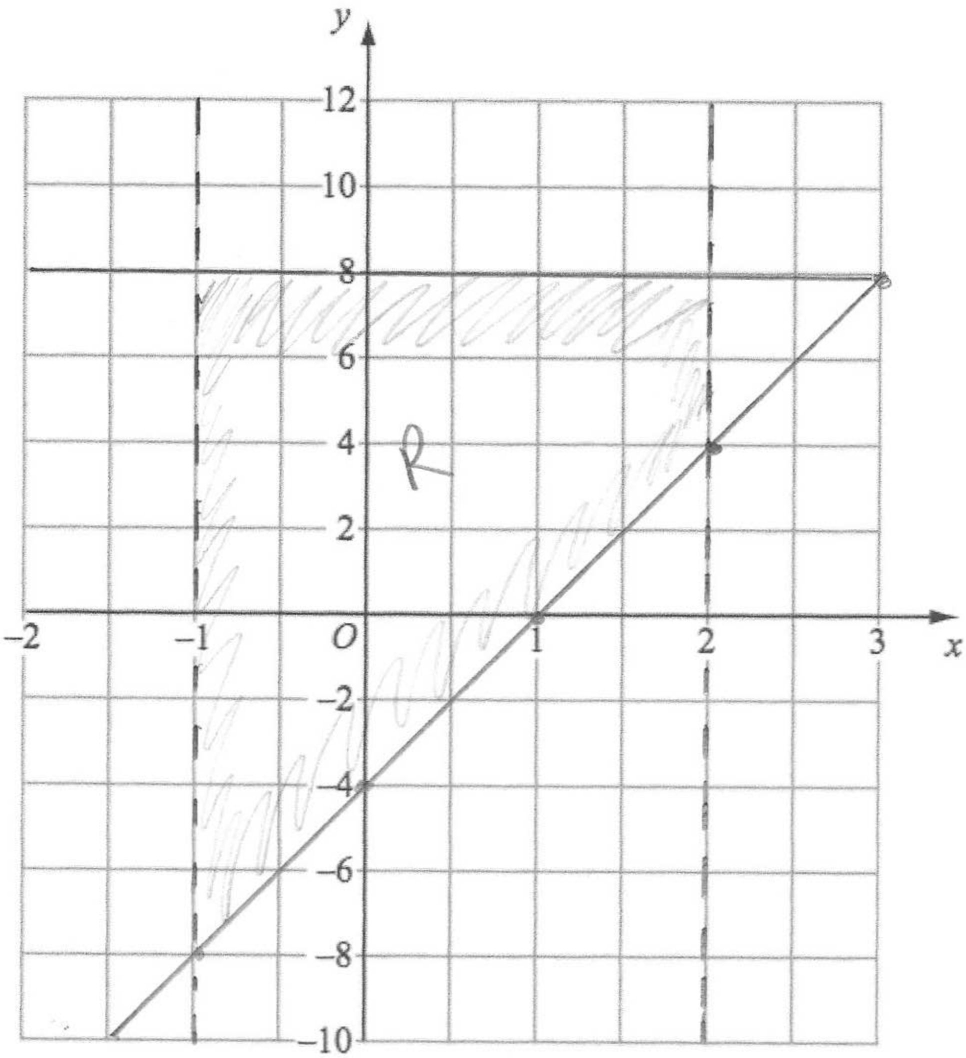


$x < 3$
(1)

(c) Write down all the integers that satisfy both inequalities shown in part (a) and (b).

$-1, 0, 1, 2$
(1)

63.



On the grid, label the region that satisfies all three of these inequalities

$-1 < x < 2$ $\qquad$ $y \leq 8$ $\qquad$ $y \geq 4x - 4$

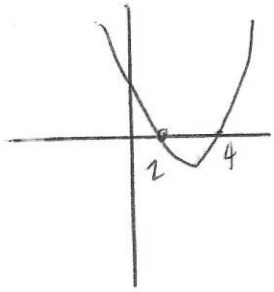
(4)

64. Solve the inequality $x^2 - 6x + 8 \geq 0$

$(x-2)(x-4) \geqslant 0$

$x \geqslant 4$

or $x \leqslant 2$

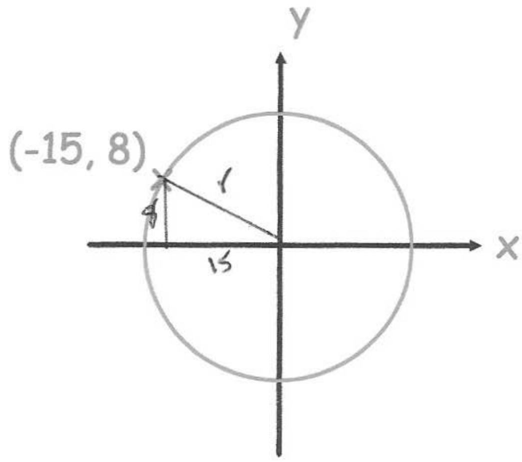


(3)

---

65. The circle below has centre (0, 0).
The point (–15, 8) is a point on the circle.

Find the equation of the circle.



$15^2 + 8^2 = r^2$

$225 + 64 = r^2$

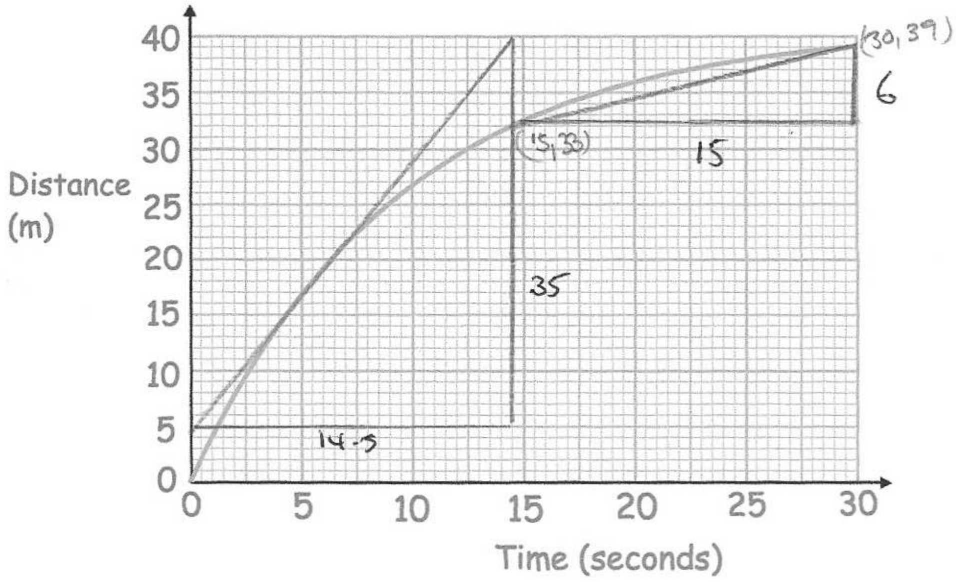
$289 = r^2$

$r = 17$

$x^2 + y^2 = 289$

(3)

66.



(a) Work out the speed at t = 5 seconds

$\frac{\text{Rise}}{\text{Run}} = \frac{35}{14.5} = 2.413\ldots$

$= 2.41$

2.41 m/s

(3)

(b) Work out the average speed between 15 and 30 seconds

$\frac{\text{Rise}}{\text{Run}} = \frac{6}{15} = 0.4$

0.4 m/s

(3)

---

67. Solve

$$\frac{3}{x-2} + \frac{3}{x+2} = 2$$

$\frac{3(x+2) + 3(x-2)}{(x-2)(x+2)} = 2$

$\frac{3x+6+3x-6}{(x-2)(x+2)} = 2$

$6x = 2(x-2)(x+2)$

$6x = 2(x^2-4)$

$6x = 2x^2 - 8$

$0 = 2x^2 - 6x - 8$

$\div 2 \quad 0 = x^2 - 3x - 4$

$(x+1)(x-4)$

$x = -1$ or $x = 4$

(5)

68. The functions $f(x)$, $g(x)$ and $h(x)$ are given by the following:

$$f(x) = x^2 - 3$$
$$g(x) = 2x + 1$$
$$h(x) = \frac{x}{2}$$

(a) Find $fg(x)$

$(2x+1)^2 - 3$

$(2x+1)(2x+1) - 3$

$= 4x^2 + 2x + 2x + 1 - 3$

$fg(x) = 4x^2 + 4x - 2$

(2)

(b) Find $gh(x)$

$2\left(\frac{x}{2}\right) + 1 = x + 1$

$gh(x) = x + 1$

(2)

(c) Find $h^{-1}(x)$

$y = \frac{x}{2}$

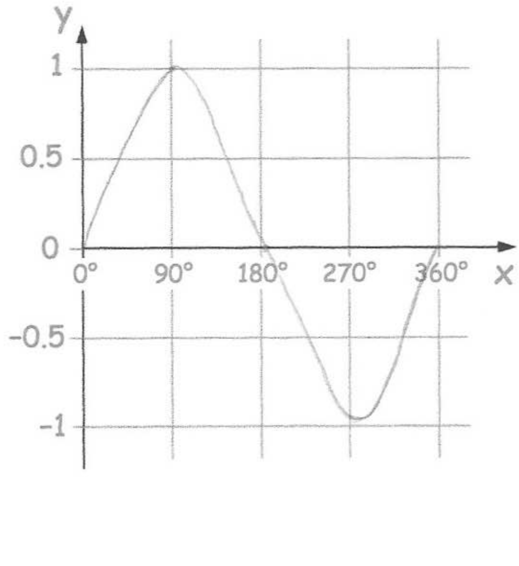
$2y = x$

$h^{-1}(x) = 2x$

(2)

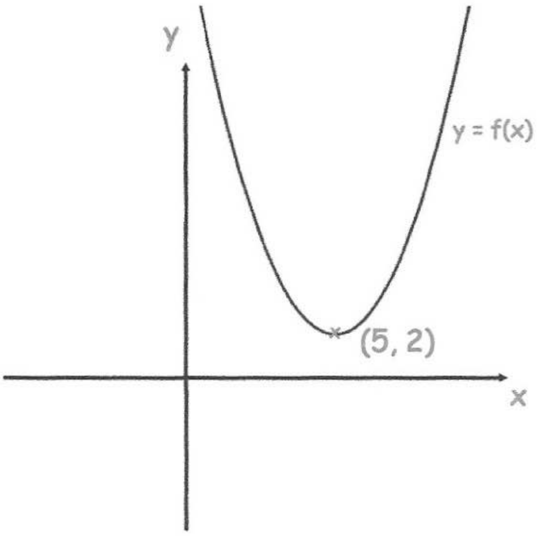
---

69. Sketch the graph of $y = sin(x)$ for $0^\circ \leq x \leq 360^\circ$



(2)

70.



Shown is the curve with equation y = f(x)
The coordinates of the minimum point of the curve are (5, 2).

Write down the coordinates of the minimum point of the curve with equation

(a) y = f(x) − 4

(......5......... , ...-2.......)
(1)

(b) y = f(x − 2)

(.........7......... , ...2.........)
(1)

(c) y = f(−x)

(.........-5......... , ...2.........)
(1)

71. Write $x^2 + 12x - 1$ in the form $(x + a)^2 + b$, where a and b are constants.

$(x+6)^2 - 36 - 1$

$(x+6)^2 - 37$

$(x+6)^2 - 37$

(3)

72. (a) Show that the equation $3x - x^3 = -11$ has a solution between $x = 2$ and $x = 3$

$3x - x^3 + 11 = 0$

when $x = 2$ $\quad 3(2) - 2^3 + 11 = 9$

$x = 3$ $\quad 3(3) - 3^3 + 11 = -7$

Since there is a change of sign between $x=2$ and $x=3$

(2)

(b) Show that the equation $3x - x^3 = -11$ can be rearranged to give

$x = \sqrt[3]{3x + 11}$

$3x + 11 = x^3$

$\sqrt[3]{3x+11} = x$

(2)

(c) Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \sqrt[3]{3x_n + 11}$ three times to find an estimate for the solution of $3x - x^3 = -11$

$x_1 = \sqrt[3]{(3 \times 3) + 11} = 2.714417617$

$x_2 = \sqrt[3]{(3 \times 2.714\ldots) + 11} = 2.675091113$

$x_3 = \sqrt[3]{(3 \times 2.675\ldots) + 11} = 2.669584272$

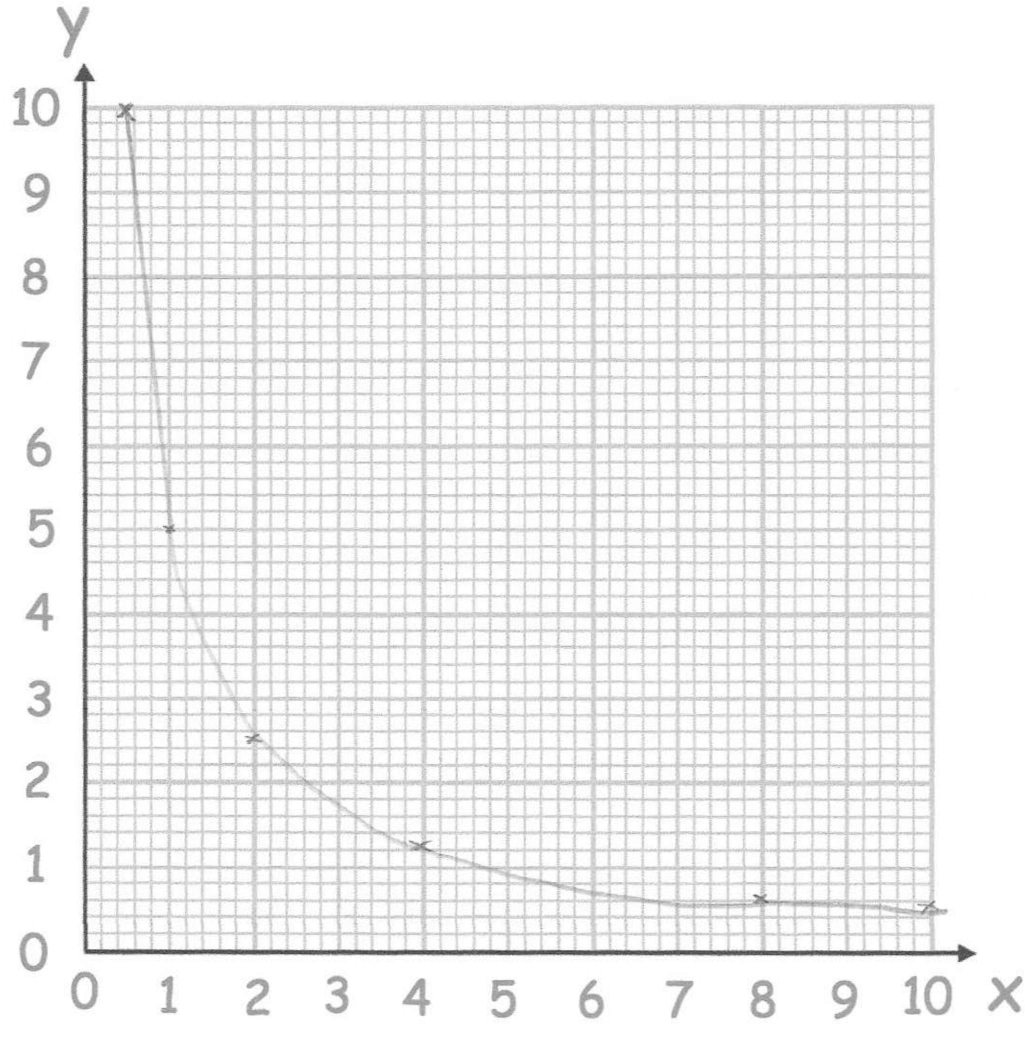
(3)

73. Complete the table of values for $y = \frac{5}{x}$

x	0.5	1	2	4	8	10
y	10	5	2.5	1.25	0.625	0.5

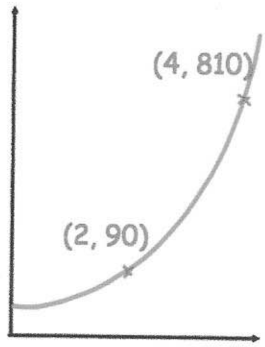
(2)

(b) On the grid, draw the graph of $y = \frac{5}{x}$ for $0.5 \leq x \leq 10$



(2)

74.



The sketch shows a curve with equation
$y = ab^x$ where a and b are constants and $b > 0$

The curve passes through the points (2, 90) and (4, 810)

Calculate the value of a and b

(2, 90)
$90 = ab^2$ — 1

(4, 810)
$810 = ab^4$ — 2

2 ÷ 1
$9 = b^2$
$b = 3$
Sub into 1
$90 = a \times 9$

a = 10

b = 3

(3)

---

75. Use algebra to prove

$$0.2\dot{8} = \frac{13}{45}$$

$\frac{26}{90} \xrightarrow{\div 2} \frac{13}{45}$

$x = 0.28888....$
$10x = 2.8888.....$
$100x = 28.888....$
$90x = 26$

(3)

76. A shed has dimensions, in metres, of

$$\text{height} = \sqrt{5}, \quad \text{width} = \sqrt{6} \text{ and length} = \frac{9}{\sqrt{2}}$$

Find the volume of the shed.
Give your answer in the form $a\sqrt{15}$, where a is an integer.

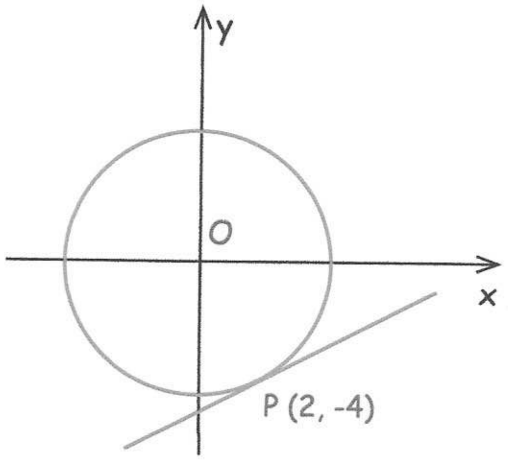
$\sqrt{5} \times \sqrt{6} \times \frac{9}{\sqrt{2}}$

$\sqrt{30} \times \frac{9}{\sqrt{2}} = \frac{9\sqrt{30}}{\sqrt{2}} = 9\sqrt{15}$

$9\sqrt{15}$ m³

(3)

77. Here is a circle, centre O, and the tangent to the circle at the point (2, –4).



Find the equation of the tangent at the point P.

Gradient OP $= -2$

$y = \frac{1}{2}x + c$

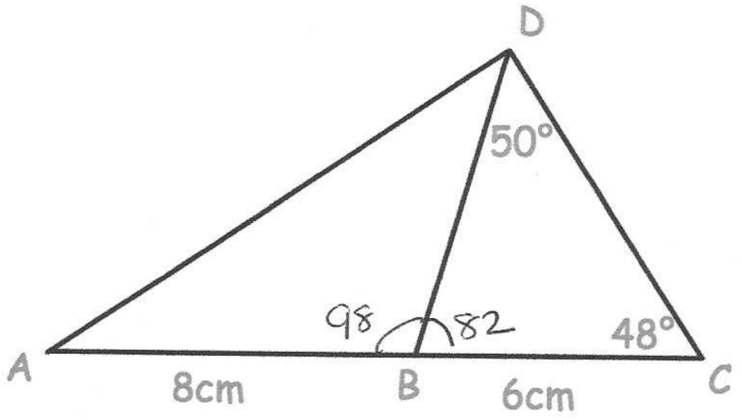
$-4 = \frac{1}{2}(2) + c$

$c = -5$

$y = \frac{1}{2}x - 5$

(3)

78.



ACD is a triangle and B is a point on AC.
AB = 8cm and BC is 6cm.
Angle BCD = 48° and angle BDC = 50°.

(a) Find the length of BD.

$$\frac{x}{\sin 48} = \frac{6}{\sin 50}$$

5.82 cm
(3)

(b) Find the length of AD.

$AD^2 = 8^2 + 5.82^2 - 2(8)(5.82)\cos 98$

$AD^2 = 110.83\ldots$

10.53 cm
(3)

(c) Find the area of triangle ABD.

$\frac{1}{2}(8)(5.82)\sin 98$

$= 23.05$

23.05 cm^2
(3)

79. A cylinder is placed on a table.
The cylinder has a weight of 400N and has a diameter of 10cm.

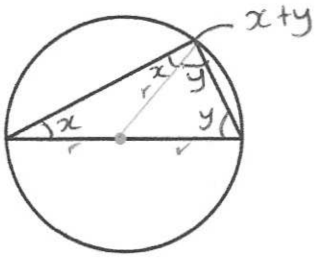
Work out the pressure on the table in newtons/cm²

$\text{Area} = \pi(5^2)$
$= 78.5398\ldots$

$P = \frac{F}{A} \quad \frac{400}{78.54} = 5.092958179$

.........5.093.....N/cm²
(3)

80.



Prove that the angle in a semi-circle is always 90°

$x + y + (x+y) = 180°$

$2x + 2y = 180°$

$x + y = 90°$

(3)

81. The mass of a paperweight is m grams.
The length of the paperweight is L centimetres.
m is directly proportional to the cube of L.

m = 4968 when L = 12

(a) Work out an equation connecting m and L

$m \propto L^3$ $\quad 4968 = K(12)^3$

$m = KL^3$ $\quad K = 23/8$

$m = \frac{23}{8}L^3$

(3)

(b) Work out the mass of a paperweight with a length of 4 centimetres

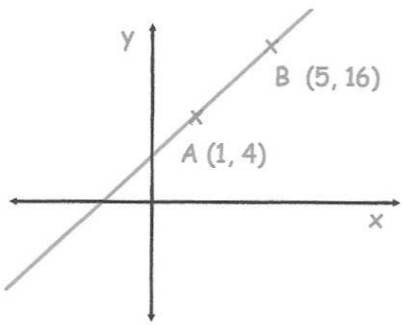
$m = \frac{23}{8}(4)^3$

184g

(2)

---

82. A straight line passes through the points A(1, 4) and B(5, 16).



(a) Find the equation of the line parallel to AB that passes through (1, 7)

$\frac{12}{4} = 3$ $\quad y = 3x + C$

$4 = 3(1) + C$ $\quad 7 = 3(1) + C$

$C = 1$ $\quad 4 = C$

$y = 3x + 4$

(2)

(b) Find the equation of the line perpendicular to AB that passes through the midpoint of AB

Midpoint $= \frac{1+5}{2}, \frac{4+16}{2}$ perpendicular gradient $= -1/3$

$= (3, 10)$ $\quad 10 = -\frac{1}{3}(3) + C$

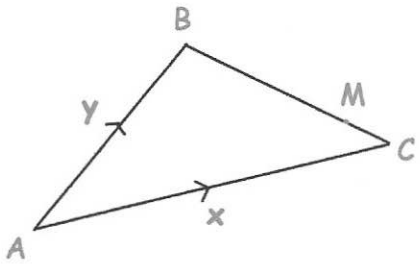
$y = -1/3x + 11$

(3)

$10 = -1 + C$

$C = 11$

83.



ABC is a triangle.
M lies on BC such that BM = $\frac{4}{5}$ BC

Express these vectors in terms of **x** and **y**

(a) $\overrightarrow{BC}$

$-\underline{y} + \underline{x}$ **(1)**

(b) $\overrightarrow{BM}$

$-\frac{4}{5}\underline{y} + \frac{4}{5}\underline{x}$ **(1)**

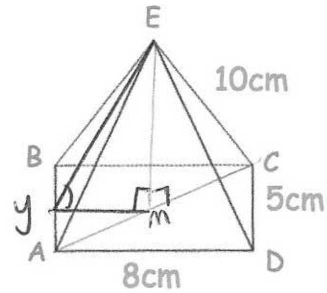
(c) $\overrightarrow{AM}$

$\underline{y} - \frac{4}{5}y + \frac{4}{5}\underline{x}$

(1)

$\frac{1}{5}\underline{y} + \frac{4}{5}\underline{x}$

84. Shown below is a rectangular based pyramid.
The apex E is directly over the centre of the base.



AD = 8cm
CD - 5cm
CE = 10cm

(a) Calculate the height of the pyramid

$AC^2 = 8^2 + 5^2$

$AC^2 = 89$

$AC = \sqrt{89}$

$MC = \frac{\sqrt{89}}{2}$

$= 4.71699...$

$10^2 - MC^2 = EM^2$

$EM^2 = 77.75$

$EM = 8.8175...$

$= 8.82$

..........8.82.......cm
(4)

(b) Calculate angle between the face ABE and the base ABCD

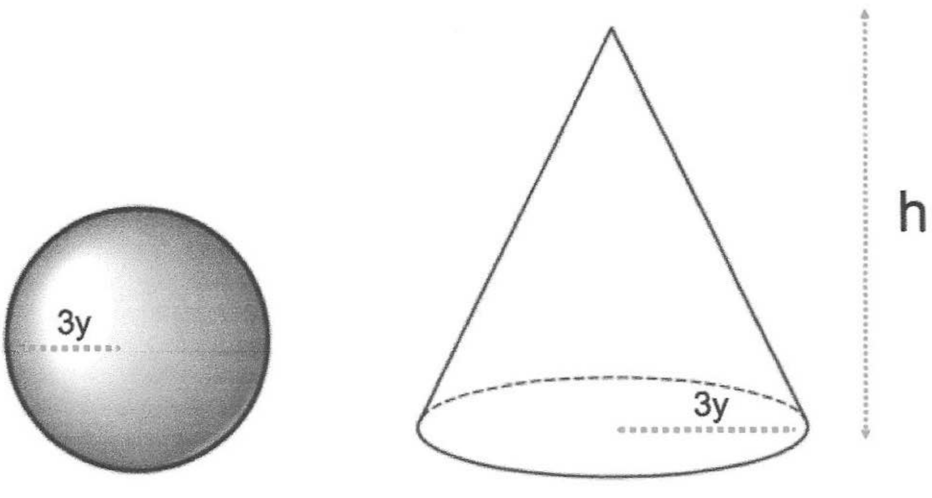
$\tan x = \frac{O}{A}$

$\tan x = \frac{8.8175...}{4}$

$\tan^{-1}\left(\frac{8.8175}{4}\right) = 65.6°$

65.6° ....°
(3)

85. This sphere and cone have the same volume.



Find an expression for h in terms of y.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi \times (3y)^3$

$V = \frac{4}{3}\pi \times 27y^3$

$V = \frac{1}{3}(\pi r^2)h$

$V = \frac{1}{3}\pi(3y)^2 \times h$

$V = \frac{1}{3}\pi \times 9y^2 \times h$

$\frac{4}{3}\pi \times 27y^3 = \frac{1}{3}\pi \times 9y^2 \times h$

$4\pi \times 27y^3 = \pi \times 9y^2 \times h$

$108y^3 = 9y^2 h$

$12y = h$

h = 12y................

(5)

86. There are 50 students in Year 11.
Each student studies one language.

	French	German
Female	13	15
Male	5	17
	18	32

Two of these students are selected at random.

Calculate the probability that the two chosen students study the same language.

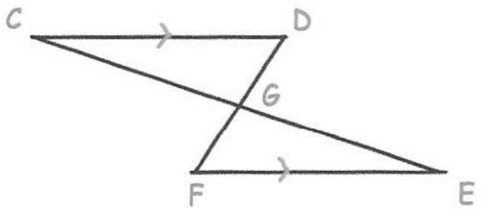
$P(FF) = \frac{18}{50} \times \frac{17}{49} = \frac{153}{1225}$

$P(GG) = \frac{32}{50} \times \frac{31}{49} = \frac{496}{1225}$

$649/1225$

(4)

87. In the diagram, the lines CE and DF intersect at G.
CD and FE are parallel and CD = FE.



Prove that triangles CDG and EFG are congruent.

CD = FE (given)

Angle DCE = FEC (alternate angles)

Angle CDF = EFD (alternate angles)

CDG and EFG are congruent as ASA

(3)

88. The first five terms of a linear sequence are 5, 11, 17, 23, 29 ...

6 6 6 6

(a) Find the nth term of the sequence

$6(1) = 6 - 1 = 5$ $\quad$ $6n-1$

$6(2) = 12 - 1 = 11$

$6n - 1$ ....

(2)

A new sequence is generated by squaring each term of the linear sequence and then adding 5.

(b) Prove that all terms in the new sequence are divisible by 6.

$(6n-1)^2 + 5$

$36n^2 - 12n + 1 + 5$

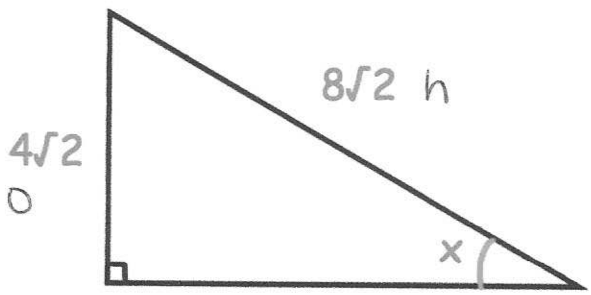
$36n^2 - 12n + 6$

$6(6n^2 - 2n + 1)$ $\therefore$ divisible by 6

(4)

---

89. Below is a right angled triangle.



Show that angle $x = 30°$
Include all your working.

$\sin x = \dfrac{4\sqrt{2}}{8\sqrt{2}} = \dfrac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$= 30°$

(2)

90. On 1st March 2001, the ratio of Freddie's age to his mother's age was 1:11
On 1st March 2018, the ratio of Freddie's age to his mother's age was 2:5

Write the ratio of Freddie's age to his mother's age on 1st March 2030

2001: $x : 11x$

2018: $x+17 : 11x+17 = 2:5$

$$\frac{x+17}{11x+17} = \frac{2}{5}$$

$5x + 85 = 22x + 34$

$51 = 17x$

$x = 3$

So in 2001, Freddie is 3 and his mother is 33.

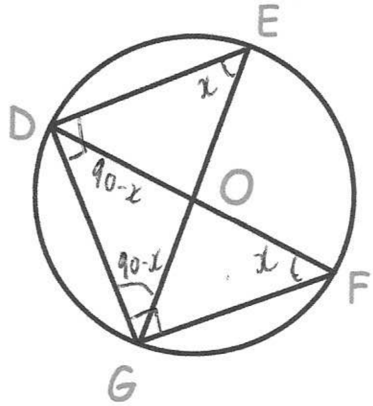
in 2030: Freddie is 32
Mother is 62

32:62 ↓÷2.

16:31

(4)

91.



O is the centre of the circle.
DOF and EOG are diameters of the circle shown.
Prove triangles DEG and DFG are congruent.

$\angle DEG = \angle DFG = x^\circ$ As angles in the same segment are equal

$\angle DGF = \angle EDG = 90^\circ$ As angle in a semi-circle is 90°

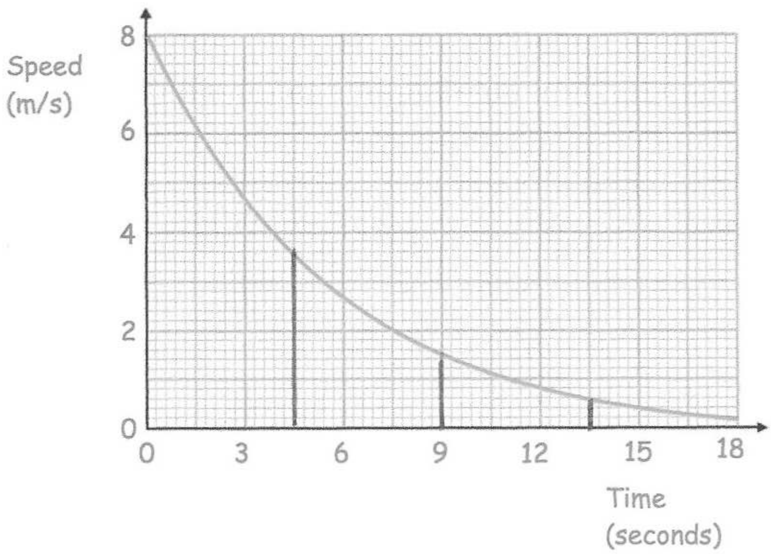
$\angle FDG = \angle EGD = 90^\circ - x$ As angles in a triangle add up to 180°

$DF = EG$ As both lines are diameters

$\therefore$ $\triangle DEG$ and $\triangle DFG$ are congruent due to ASA.

(3)

92. Here is a speed-time graph for a remote-controlled car



(a) Work out an estimate for the distance travelled over the first 12 seconds of the journey.
Use 4 strips of equal width.

$\frac{1}{2}(a+b)\times h$

① $\frac{1}{2}(8+3.5)\times 4.5 = 25.875$

④ $\frac{1}{2}(0.6+0.2)\times 4.5 = 1.8$

② $\frac{1}{2}(3.5+1.5)\times 4.5 = 11.25$

③ $\frac{1}{2}(1.5+0.6)\times 4.5 = 4.725$

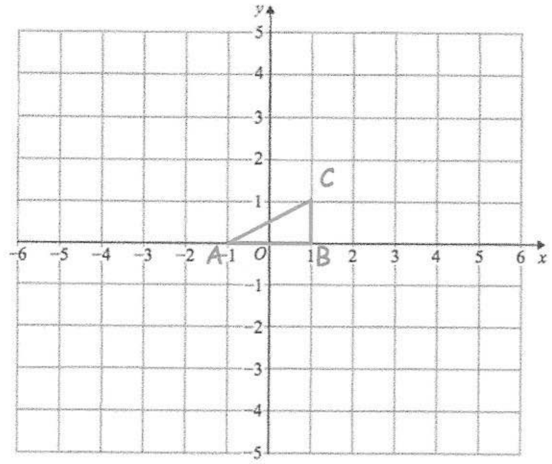
43.65 m
(4)

(b) Is your answer to (a) an overestimate or an underestimate of the actual distance travelled?
Explain your answer

Overestimate as each trapezium is over the actual curve so the area will be slighty less than what has been used.
(1)

93. Shown is triangle ABC

ABC is rotated 180° about $(-1, 2)$ and then translated by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$



Write down the coordinate of the invariant point.

(0, 0)

(3)

94. Solve the equations

$$y = x^2 - 5$$
$$y = 2x - 2$$

$x^2 - 5 = 2x - 2$

$x^2 - 2x - 5 + 2 = 0$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = 3 \quad x = -1$

$y = 4 \quad y = -4$

$x = 3$ or $x = -1$

$y = 4 \quad y = -4$ **(5)**

$(3, 4)$ & $(-1, -4)$