Write your name here

Surname	Other Names

Mathematics

2019 Practice Paper
Paper 3 (Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- **Calculators may be used.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**

Information

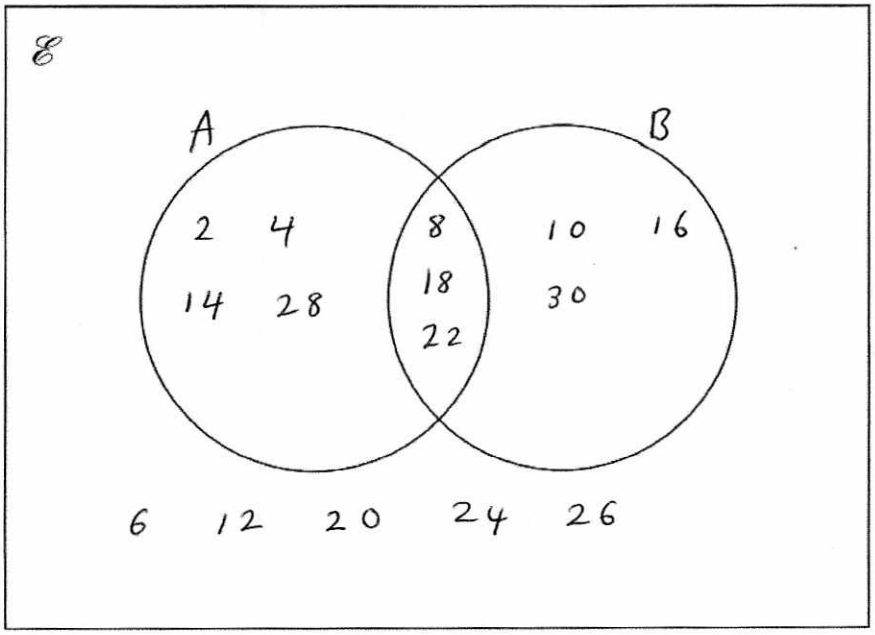
- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

1 $\mathscr{E}$ = {even numbers between 1 and 31}
A = {2, 4, 8, 14, 18, 22, 28}
B = {8, 10, 16, 18, 22, 30}

(a) Complete the Venn diagram to represent this information.



(4)

A number is chosen at random from the universal set, $\mathscr{E}$

(b) What is the probability that the number is in the set $A \cup B$?

$\frac{10}{15}$ [or $\frac{2}{3}$]

.....

(2)

(Total for question 1 is 6 marks)

2 The frequency table shows the time taken for 100 people to travel to an event.

Time (minutes)	Frequency
$0 < t \leqslant 10$	14
$10 < t \leqslant 20$	16
$20 < t \leqslant 30$	23
$30 < t \leqslant 40$	29
$40 < t \leqslant 50$	12
$50 < t \leqslant 60$	6

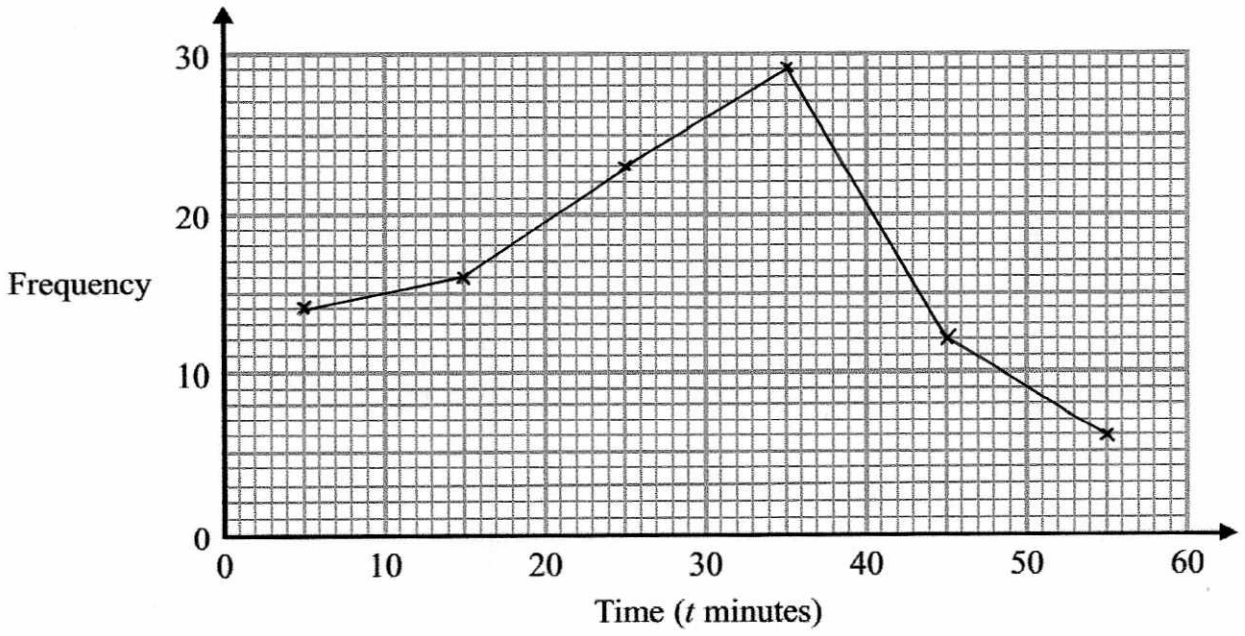
(a) Find the percentage of people that travelled for more than 30 minutes to the event

29 + 12 + 6

47 %

1 ~~(2)~~

(b) Draw a frequency polygon for the information on the table.



(2)

(Total for question 2 is 4 marks)

3 (a) Find the reciprocal of 8

$\frac{1}{8}$

.....

(1)

(b) Use your calculator to work out $(2 \cos 40° + 3 \sin 25°)^3$
Write down all the figures on your calculator display.

21.950675 18

.....

(2)

(Total for question 3 is 3 marks)

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4 Solve the simultaneous equations

$2x + 5y = 2$ x 7
$7x - 4y = -1$ x 2

$14x + 35y = 14$
$14x - 8y = -2$

$43y = 16$

$y = \frac{16}{43}$

$2x + 5\left(\frac{16}{43}\right) = 2$

$2x + \frac{80}{43} = 2$

$2x = \frac{6}{43}$

$x = \frac{3}{43}$

$x =$ $\frac{3}{43}$

$y =$ $\frac{16}{43}$

(Total for question 4 is 3 marks)

x_1 y_1

5 A is the point with coordinates (3, 8)
B is the point with coordinates (x, 13)

x_2 y_2

The gradient of AB is 2.5
Work out the value of x

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2.5 = \frac{13 - 8}{x - 3}$$

$$2.5 = \frac{5}{x - 3}$$

$$x - 3 = 2$$

$$x = 5$$

5

(Total for question 5 is 2 marks)

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6 (a) Olivia is going to invest some money for 5 years.

She can choose from two options:

Investment A: 2.7% compound interest per annum

Investment B: 2.8% simple interest per annum

Which investment should Olivia choose
You must show your working.

A

100×1.027^5

$= 114.24895022$

14.25% Increase

B

$2.8 \times 5 = 14$

14% Increase

Investment A

(Total for question 6 is 4 marks)

7 The exchange rate in London is £1 = $1.31

The exchange rate in New York is $1 = £0.79

Bernie wants to change some pounds into dollars.

In which of these cities would Bernie get the most dollars?
You must show your working.

New York $1 = £0.79
÷0.79 ÷0.79
$1.27 = £1

London £1 = $1.31

LONDON

3
(Total for question 7 is ~~2~~ marks)

8 Each year Rose buys an annual ticket for his train journey to work.

The price of Rose's ticket increased by 2% in 2017 and 3% in 2018.

The ticket cost £2534 in 2018.

What was the price of the ticket in 2016?

$$x \times 1.02 \times 1.03 = 2534$$

$$x = \frac{2534}{1.02 \times 1.03}$$

£ 2411.96

(Total for question 8 is 3 marks)

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9 Last year Patrick paid £2534 for his annual train ticket.
This year he has to pay £2612 for his annual train ticket.

Work out the percentage increase in the cost of his train ticket.
Give your answer correct to 3 significant figures

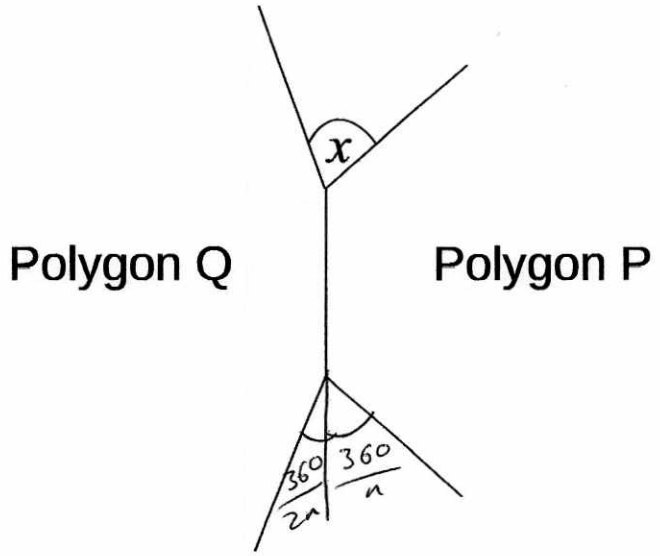
$$\frac{\text{change}}{\text{original}} \times 100$$

$$\frac{2612 - 2534}{2534} \times 100$$

3.08 %

(Total for question 9 is 3 marks)

10 Two regular polygons P and Q have a common side as shown in the diagram.



Polygon P has n sides. Polygon Q has twice as many sides as Polygon P.

Find the size of angle x in terms of n.

2n sides

Exterior Angle of P $= \frac{360}{n}$

Exterior Angle of Q $= \frac{360}{2n}$

$$x = \frac{360}{n} + \frac{360}{2n}$$

$$= \frac{360}{n} + \frac{180}{n}$$

$$= \frac{540}{n}$$

$\frac{540}{n}$

.....

(Total for question 10 is 3 marks)

11 Liquid **A** has a density of 1.2 g/cm^3

150 cm^3 of Liquid **A** is mixed with some of Liquid **B** to make Liquid **C**.

Liquid **C** has a mass of 210 g and a density of 1.12 g/cm^3

Density = $\frac{\text{Mass}}{\text{Volume}}$

Find the density of Liquid **B**.

~~Give your answer to 2 decimal places.~~

A// mass = density × volume

= 1.2 × 150

= 180g

C// volume = $\frac{\text{mass}}{\text{density}}$

= $\frac{210}{1.12}$

= 187.5

B// mass = 210 − 180 = 30g

volume = 187.5 − 150 = 37.5 cm^3

density = $\frac{30}{37.5}$

= 0.8

0.8 g/cm^3

(Total for question 11 is 3 marks)

12 Emma has a bag containing a large number of beads.
She wants to find an estimate for the number of beads in the bag.

Emma takes a sample of 50 beads from the bag.
She marks each bead with a black cross and then puts the beads back in the bag.

Emma shakes the bag.
She now takes another sample of 50 beads from the bag.

6 of these beads have been marked with a black cross.

Work out an estimate for the total number of beads in the bag.

$$\frac{50}{x} = \frac{6}{50}$$

$\frac{50}{6} = 8.\dot{3}$ $\quad 50 \times 8.\dot{3} = 416.\dot{6}$

417

(Total for question 12 is 2 marks)

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13 A radioactive substance decays by x % each day. After 8 days half of the substance has decayed.
Find the value of x.
Give your answer to 1 decimal place.

$$y^8 = 0.5$$
$$y = \sqrt[8]{0.5}$$
$$y = 0.917$$

$(1 - 0.917) \times 100 = 8.3\%$

8.3

(Total for question 13 is 3 marks)

14 (a) Expand and simplify $(x + 5)(x + 3)(x - 4)$

$(x+5)(x+3)(x-4)$

$(x+5)(x^2-4x+3x-12)$

$(x+5)(x^2-x-12)$

$x^3-x^2-12x+5x^2-5x-60$

$x^3+4x^2-17x-60$

$x^3+4x^2-17x-60$

(3)

(b) Solve $3x^2 - 5x - 7 = 0$
Give your solutions correct to 3 significant figures

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2-4(3)(-7)}}{2(3)}$$

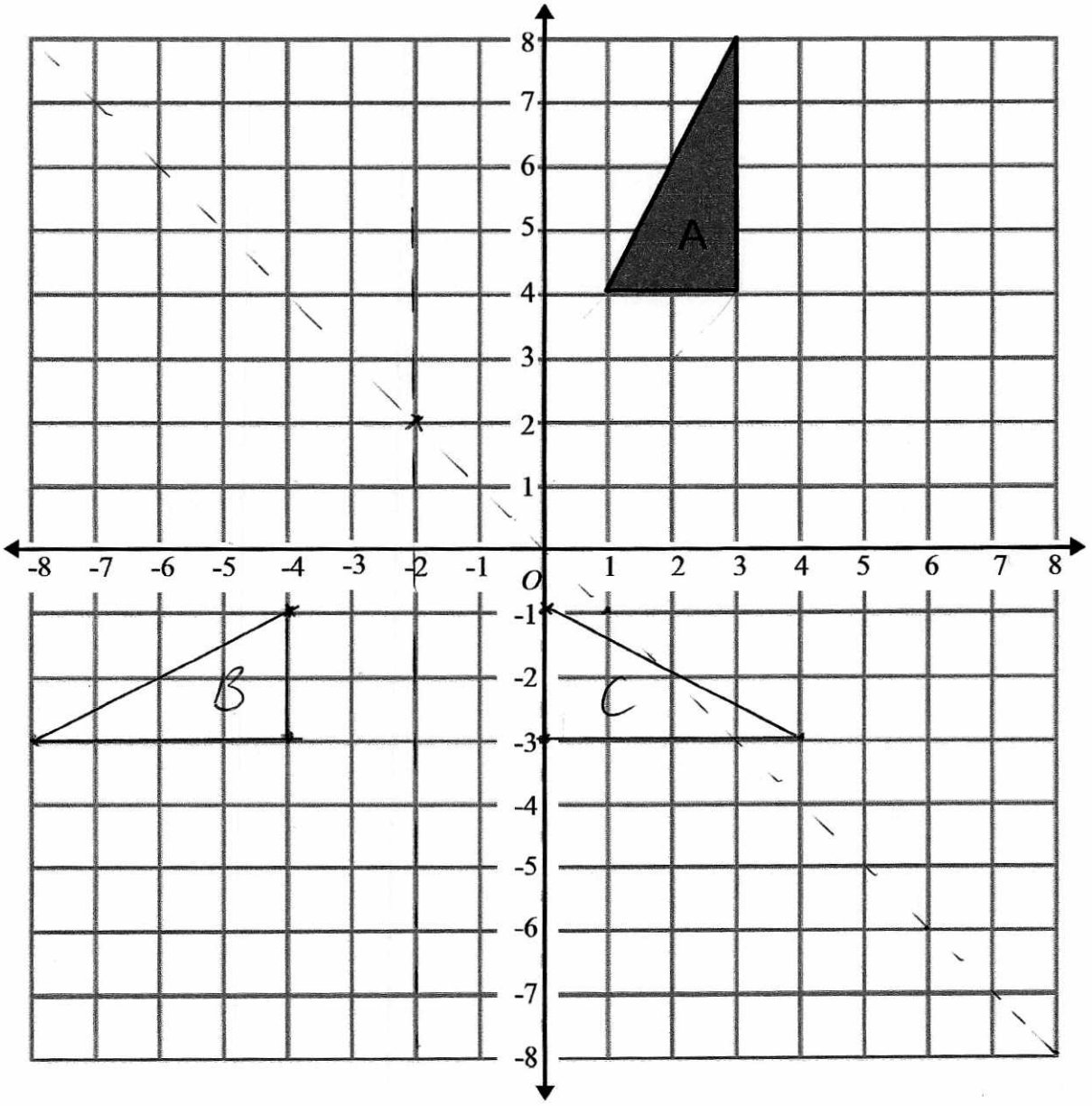
$x = 2.57$ and $x = -0.907$

2.57 and -0.907

(3)

(Total for question 14 is 6 marks)

15



Triangle A is reflected in the line $y = -x$ to give triangle B

Triangle B is reflected in the line $x = -2$ to give triangle C

Describe the single transformation which maps triangle A onto triangle C.

Rotation 90° Clockwise centre (−2, 2)

(Total for question 15 is 3 marks)

16 Prove algebraically that the recurring decimal $0.\dot{3}\dot{1}\dot{5}$ can be written as $\frac{35}{111}$

$0.\dot{3}1\dot{5} = x$

$315.\dot{3}1\dot{5} = 1000x$

$315 = 999x$

$x = \frac{315}{999} = \frac{35}{111}$

(Total for question 16 is 2 marks)

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17 Here are the first 5 terms of a quadratic sequence.

$an^2 + bn + c$

5	11	22	38	59

Find an expression, in terms of n, for the nth term of this sequence.

5 11 22 38

$a+b+c$ → 6 11 16

$3a+b$ → 5 5

$2a$

$2a = 5$

$a = 2.5$

$3a + b = 6$

$3(2.5) + b = 6$

$7.5 + b = 6$

$b = -1.5$

$a + b + c = 5$

$2.5 - 1.5 + c = 5$

$1 + c = 5$

$c = 4$

$2.5n^2 - 1.5n + 4$

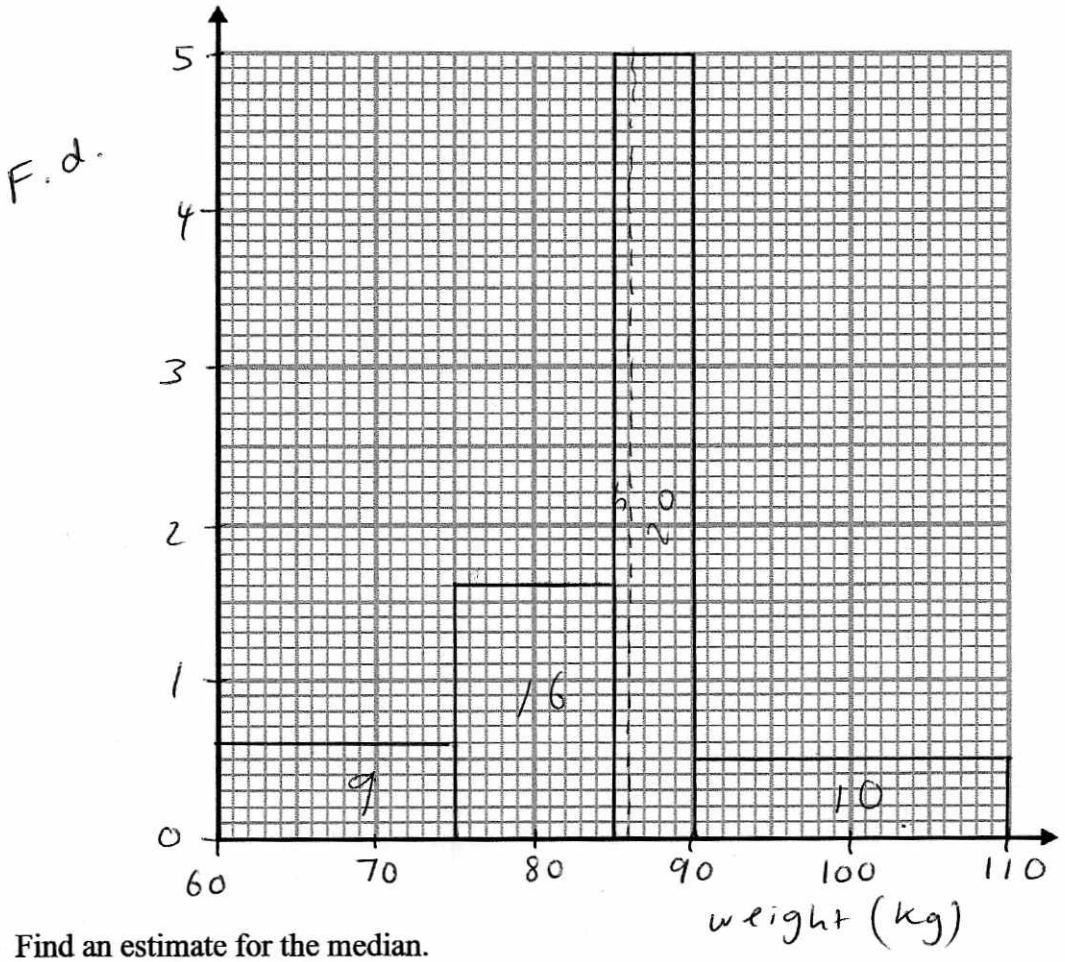
(Total for question 17 is 3 marks)

18 The table shows information about the weight of 60 pigs.

Weight (kg)	Frequency
$60 < w \leqslant 75$	9
$75 < w \leqslant 85$	16
$85 < w \leqslant 90$	25
$90 < w \leqslant 110$	10

F.d
0.6
1.6
5
0.5

(a) On the grid, draw a histogram for the information in the table.



(3)

(b) Find an estimate for the median.

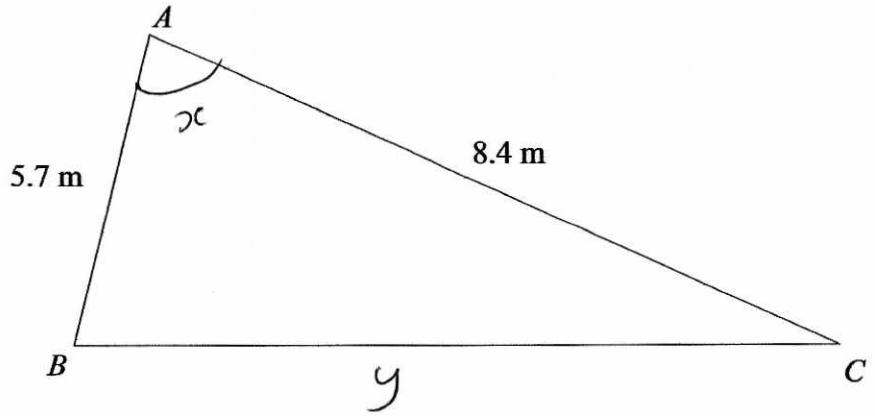
30 on each side

86 .....kg

(2)

(Total for question 18 is 5 marks)

19



The area of the triangle is $21m^2$
Calculate the perimeter of triangle *ABC*.
Give your answer to 1 decimal place.

$\frac{1}{2}\, a\, b \sin C = 21$

$\frac{1}{2}(5.7)(8.4)\sin x = 21$

$\sin x = \frac{50}{57}$

$x = 61.30558647$

$a^2 = b^2 + c^2 - 2bc\cos A$

$y^2 = (5.7)^2 + (8.4)^2 - 2(5.7)(8.4)\cos(\text{"}61.3\text{"})$

$y^2 = 57.07198786$

$y = 7.5546\ldots$

Perimeter $= 5.7 + 8.4 + 7.5546$

$= 21.7\ m$

21.7 .....m

(Total for question 19 is 5 marks)

20 (a) Show that the equation $x^3 - 4x^2 + 1 = 0$ has a solution between $x = 3$ and $x = 4$

$(3)^3 - 4(3)^2 + 1 = -8$

$(4)^3 - 4(4)^2 + 1 = 1$

change of sign $\therefore$ solution between $x = 3$ and $x = 4$

(2)

(b) Show that the equation $x^3 - 4x^2 + 1 = 0$ can be rearranged to give: $x = \sqrt[3]{4x^2 - 1}$

$x^3 - 4x^2 + 1 = 0$

$x^3 + 1 = 4x^2$

$x^3 = 4x^2 - 1$

$x = \sqrt[3]{4x^2 - 1}$

(1)

(c) Starting with $x_0 = 4$, use the iteration formula $x_{n+1} = \sqrt[3]{4x_n^2 - 1}$ to find the value of x_2

Give your answer to 3 decimal places.

$x_1 = \sqrt[3]{4(4)^2 - 1}$

$= 3.979057208$

$x_2 = \sqrt[3]{4(\text{Ans})^2 - 1}$

$= 3.964934863$

3.965

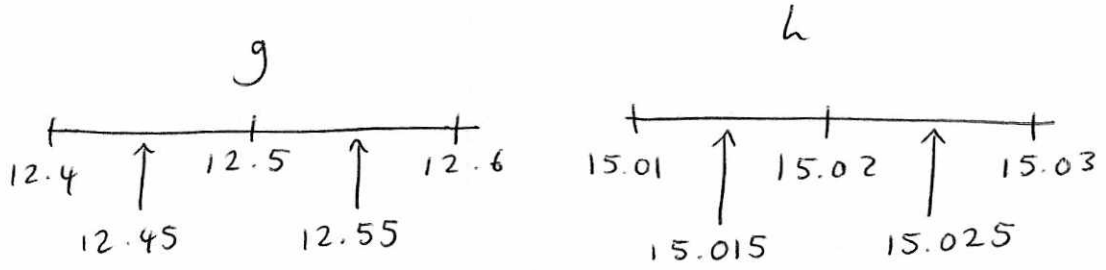
(3)

(Total for question 20 is 6 marks)

21 $f = \dfrac{\sqrt{g}}{h}$

g = 12.5 correct to 3 significant figures
h = 15.02 correct to 4 significant figures

By considering bounds, work out the value of f to a suitable degree of accuracy.
Give a reason for your answer.



upper f $= \dfrac{\sqrt{\text{upper } g}}{\text{lower } h}$ $= \dfrac{\sqrt{12.55}}{15.015}$ $= 0.235937\ldots$

lower f $= \dfrac{\sqrt{\text{lower } g}}{\text{upper } h}$ $= \dfrac{\sqrt{12.45}}{15.025}$ $= 0.234838\ldots$

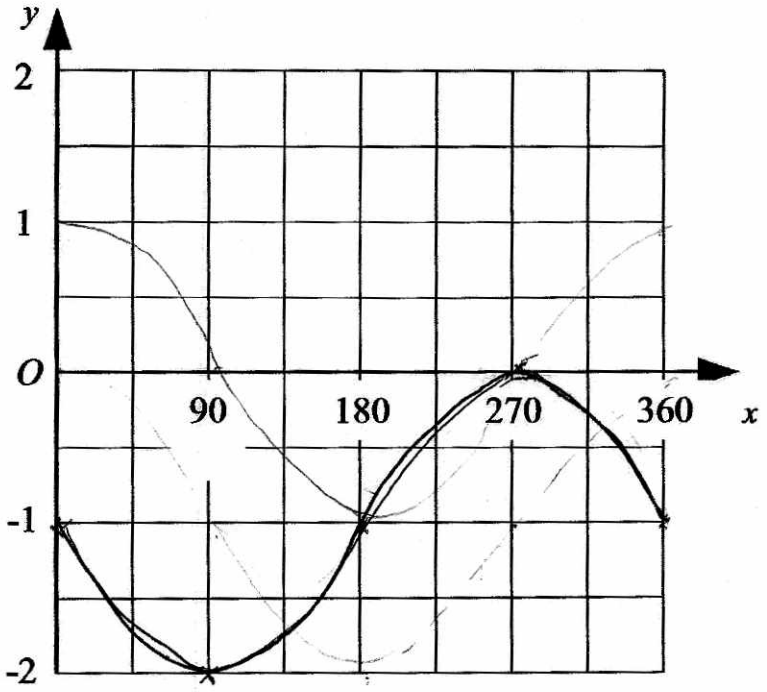
$f = 0.2$ (1dp)

Both round to 0.2 to 1dp

0.2

(Total for question 21 is 5 marks)

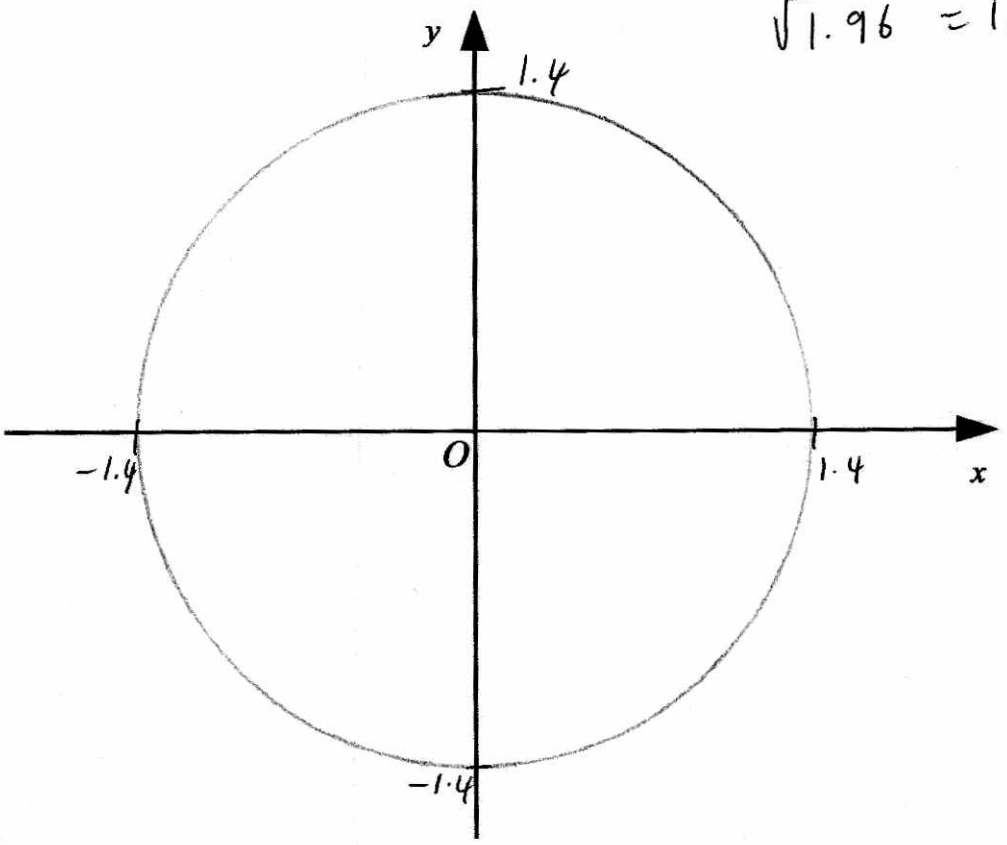
22 (a) On the graph draw a sketch of $y = \cos(x + 90) - 1$ from $0° \leq x \leq 360°$



(2)

(b) Sketch the graph of $x^2 + y^2 = 1.96$

$\sqrt{1.96} = 1.4$



(2)

(Total for question 22 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS