

S66513A

- Check your answers if you have time at the end.
- Try to answer every question.
- Read each question carefully before you start to answer it.

Advice

- The total mark for this paper is 80
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.

Information

- Calculators may be used.
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must show all your working.
- there may be more space than you need.
- Answer the questions in the spaces provided.
- Answer all questions.
- and candidate number.



- Fill in the boxes at the top of this page with your name, centre number
- Use black ink or ball-point pen.

Instructions

Please check the examination details below before entering your candidate information	
Candidate surname	Other names
Centre Number	
Candidate Number	
LPG5 Autumn Mock Exam 2020	
Time: 1 hour 30 minutes	Paper Reference 1MA1/3H
Mathematics Paper 3 (Calculator) Higher Tier	
You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.	
Total Marks	

WORKED SOLUTIONS

Answer ALL questions.

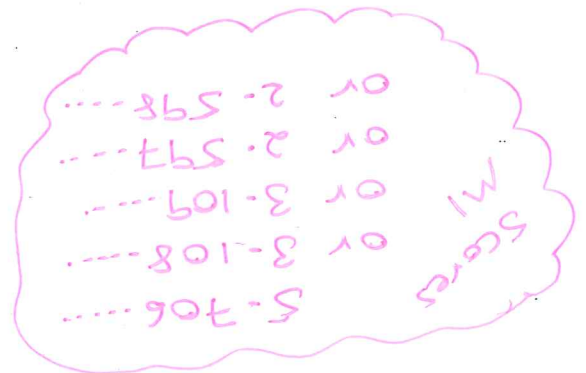
Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Calculate $\sqrt{6 \sin 72^\circ - 4 \cos 39^\circ}$

Give your answer correct to 3 significant figures.

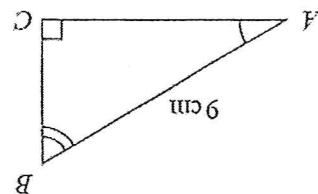
$$1.61175533 \dots = 1.61 \text{ (3s.f.)}$$



$$M1 \quad A1 \quad 1.61$$

(Total for Question 1 is 2 marks)

ABC and PQR are similar triangles.



$$AB : PQ = 2 : 5$$

(i) Work out the length of PQ .

$$AB : PQ$$

$$2 : 5$$

$$9 : 22.5$$

$$\times 4.5$$

method to find a scale factor

M1

(2)

cm

22.5

(2)

cm

7.2

$$BC : QR$$

$$2 : 5$$

$$7.2 : 18$$

$$\times 3.6$$

M1

(Total for Question 2 is 4 marks)

The table shows information about the distances travelled by 50 new cars before a tyre was changed.

Distance (d km)	Number of cars
$5000 \leq d < 25\,000$	9
$25\,000 \leq d < 45\,000$	25
$45\,000 \leq d < 65\,000$	16

Calculate an estimate for the mean distance.

$$9 \times 15000 + 25 \times 35000 + 16 \times 55000 = 37\,800$$

midpoint

15000

35000

55000

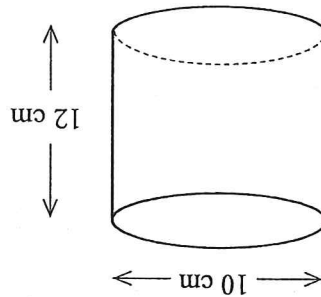
m1 (find 3 products within intervals)

50

37,800 km

(Total for Question 3 is 3 marks)

The diagram shows a cylinder with diameter 10 cm and height 12 cm.



Calculate the volume of this cylinder.

Give your answer correct to 3 significant figures.

$$\text{Volume} = \pi r^2 h$$

$$= \pi \times 5^2 \times 12$$

$$= 942.477 \dots$$

$$= 942 \text{ cm}^3 \text{ (3 s.f.)}$$

942 cm³

A1

m1 complete method

(Total for Question 4 is 2 marks)

Luke invested £4000 in a savings account for 3 years.
Compound interest was paid at a rate of 1.8% each year.

Alexa also invested £4000 in a savings account for 3 years.
Simple interest was paid at a rate of 1.8% each year.

Luke got more interest than Alexa in total over the 3 years.

How much more?

You must show all your working.

$$\text{Luke: } 4000 \times 1.018^3 = 4219.91$$

$$\text{Alexa: } 1.8\% \text{ of } 4000 = 72$$

$$4000 + 72 \times 3 = 4216$$

$$4219.91 - 4216 = 3.91$$

P1 finds 1.8% of 4000
P1 complete process
to deal with
compound interest

P1 complete process
to deal with
simple interest.

#1 (or 3.92)

£3.91

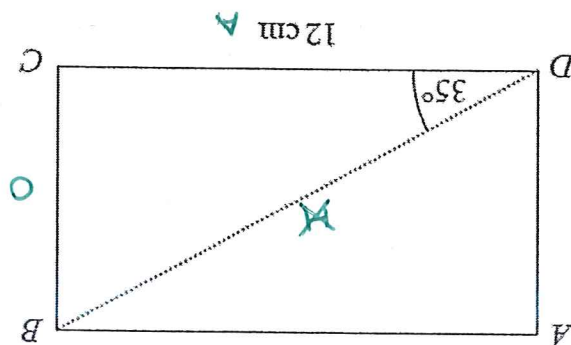
(Total for Question 5 is 4 marks)

The height, h metres, of a tall building is 184 metres correct to the nearest metre. Complete the following statement to show the range of possible values of h .

$$183.5 \leq h < 184.5$$

(Total for Question 6 is 2 marks)

Here is a rectangle $ABCD$.



Work out the perimeter of the rectangle.
Give your answer correct to 3 significant figures.

$$P1 \quad O = T \times A$$

$$BC = \tan 35^\circ \times 12 = 8.4024 \dots$$

$$P1 \quad \text{Perimeter} = 2 \times 12 + 2 \times 8.4024 \dots$$

$$= 40.8048 \dots$$

$$= 40.8 \text{ cm (3 s.f.)}$$

A1

$$40.8 \dots \text{ cm}$$

(Total for Question 7 is 4 marks)

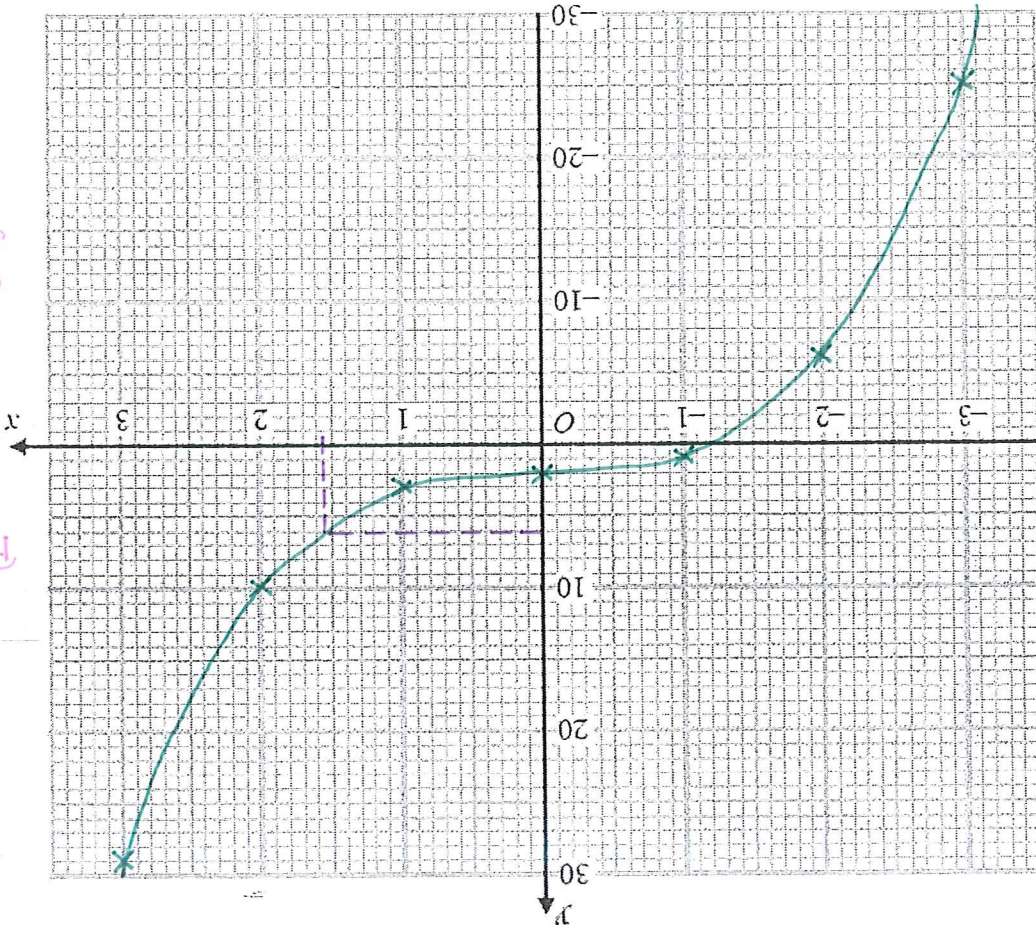
8 (a) Complete the table of values for $y = x^3 + 2$

x	y
-3	-25
-2	-6
-1	1
0	2
1	3
2	10
3	29

B1 3 or 4 correct
B1 all correct

(2)

(b) On the grid, draw the graph of $y = x^3 + 2$ for values of x from -3 to 3



M1
"values"
plotting"
(dep B1)
A1
Correct
Smooth
Curve

(c) Use your graph to find the value of x when $y = 6$

(2)

(1)

1.55

B1

1.5 → 1.7
f(1.6)

(Total for Question 8 is 5 marks)

9 Here is some information about the population and the land area of England, Scotland and Wales in 2016

	Population	Land area (km ²)
England	5.5×10^7	1.3×10^5
Scotland	5.4×10^6	8.0×10^4
Wales	1.9×10^6	2.1×10^4

(a) Calculate the total population of England, Scotland and Wales in 2016

$$62,300,000 = 6.23 \times 10^7$$

either form ok

81

(1)

$$6.23 \times 10^7$$

(b) Calculate the average number of people per km² in England, Scotland and Wales in 2016
Give your answer correct to the nearest whole number.

$$\text{Total area} = 231,000 = 2.31 \times 10^5$$

PI gives to find total area

$$\text{Average} = \frac{6.23 \times 10^7}{2.31 \times 10^5} = 269.69 \dots$$

PI

$$= 270 \text{ (3 s.f.)}$$

$$\begin{array}{r} 270 \\ 269 \rightarrow 270 \end{array}$$

PI

(3)

(Total for Question 9 is 4 marks)

(Total for Question 11 is 3 marks)

$x = 4.8$

71

M1
(dep m1) isolating term in x

$$\begin{aligned} 8 - 3x &= 32 - 8x & \times 4 \\ 8 + 5x &= 32 & + 8x \\ 5x &= 24 & - 8 \\ x &= 4.8 & \div 5 \end{aligned}$$

11 Solve $\frac{8 - 3x}{4} = 8 - 2x$

(Total for Question 10 is 2 marks)

135

71 135

$$\begin{aligned} \frac{54}{8} &= \frac{n}{20} \\ 54 &= \frac{8n}{20} & \times n \\ 1080 &= 8n & \times 20 \\ 135 &= n & \div 8 \end{aligned}$$

Work out an estimate for the value of n .

- 10 There are n adults in a club.
54 of the adults are over 30 years of age.
20 of the adults in the club are chosen at random.
8 of these 20 adults are over 30 years of age.

P1

$$\begin{aligned} \frac{54}{8} &= \frac{n}{20} & \text{or} \\ \frac{54}{8} &= \frac{n}{20} & \times 20 \\ 135 &= n & \times 20 \end{aligned}$$

She can choose from 16 girls and 14 boys.

- $$422 = 41 \times 91$$

224

- $$16 = \frac{2}{14 \times 13}$$

14

(Total for Question 12 is 3 marks)

(7)

$$\frac{n \text{ boys } (n \text{ boys} - 1)}{2}$$

(Total for Question 13 is 4 marks)

(1)

1.414 (3dp)

B1

$$x_0 = 1$$

$$x_1 = \sqrt[4]{3 \times 1 + 1} = 1.4142 \dots$$

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \sqrt[4]{3x_n + 1}$ once to find an estimate for a solution of $x^4 - 3x - 1 = 0$

(1)

$$x^4 - 3x - 1 = 0$$

$$x^4 = 3x + 1$$

$$x = \sqrt[4]{3x + 1}$$

C1

(b) Show that for $x > 0$ the equation $x^4 - 3x - 1 = 0$ can be arranged to give $x = \sqrt[4]{3x + 1}$

(2)

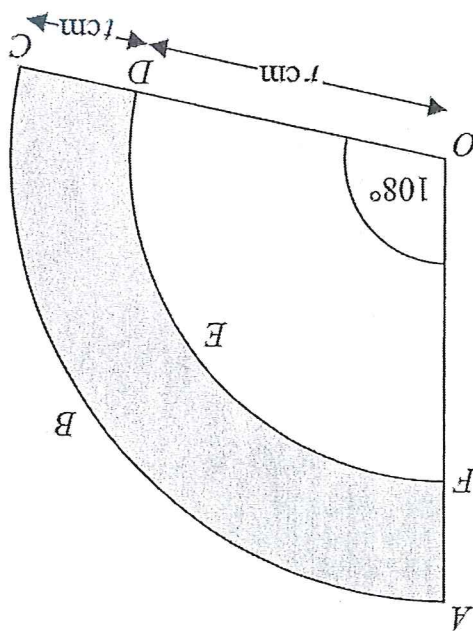
$$x = 1: 1^4 - 3 \times 1 - 1 = -3$$

$$x = 2: 2^4 - 3 \times 2 - 1 = 9$$

change of sign means there is a solution to $x^4 - 3x - 1 = 0$ between $x = 1$ and $x = 2$. C1

13 (a) Show that the equation $x^4 - 3x - 1 = 0$ has a solution between $x = 1$ and $x = 2$

ABC and DEF are two arcs of circles, centre O .
 OFA and ODC are straight lines.



(a) Show that the perimeter of the shaded region is given by $\frac{6\pi r + 3\pi t + 10t}{5}$ cm

Arc length = $\frac{\theta}{360} \times 2\pi r$

PI use of $\frac{108}{360}$

or $2\pi r$
 or $2\pi(r+t)$

length $ABC = \frac{108}{360} \times \pi \times 2(r+t)$

$= \frac{5}{3} \pi r$

length $DEF = \frac{108}{360} \times \pi \times 2t$

$= \frac{5}{3} \pi(r+t)$

Perimeter = $\frac{5}{3} \pi r + \frac{5}{3} \pi(r+t) + 2t$

$= \frac{5}{3\pi r + 3\pi r + 3\pi t + 10t}$

$= \frac{5}{6\pi r + 3\pi t + 10t}$

(5) CI completion of
 processes to
 get result.

PI collect like
 terms.

a common
 denominator

PI process to use

PI complete
 process to find
 perimeter

(Total for Question 14 is 7 marks)

(i)

The perimeter of a sector, radius 10, angle 108° .

(ii) Explain what your value in part (b)(i) represents.

(i)

$$6\pi + 20$$

$$\frac{3\pi \times 10 + 10 \times 10}{5} = 6\pi + 20$$

Give your answer in its simplest form.

(b)(i) Find the exact value of $\frac{6\pi r + 3\pi t + 10t}{5}$ when $r = 0$ and $t = 10$

71

81

15 c is inversely proportional to d
 $c = 0.5$ when $d = 6$

Find a formula for c in terms of d .

$$c \propto \frac{1}{d}$$

$$c = \frac{k}{d}$$

$$0.5 = \frac{k}{6}$$

$$3 = k$$

$$c = \frac{3}{d}$$

(Total for Question 15 is 3 marks)

16

There are some counters in a box.

Each counter is blue or green or red or yellow.

The total number of blue and green counters is twice the total number of red and yellow counters.
 The number of green counters is $\frac{1}{6}$ of the number of blue counters.

Show that, to the nearest percent, the percentage of blue counters in the box is 57%

$$b + g = 2(r + y)$$

$$g = \frac{1}{6}b$$

$\frac{3}{2}$ of counters are blue or green.

$\frac{7}{6}$ of blue or green counters are blue.

$$\frac{3}{2} \times \frac{7}{6} = \frac{7}{4} = 57.14\% \approx 57\%$$

= 57% (nearest percent).

(Total for Question 16 is 4 marks)

OR Let there be 2100 counters

Band G: 700

2 : 1

1400 : 700

now share the 1400 B and G counters

B : G

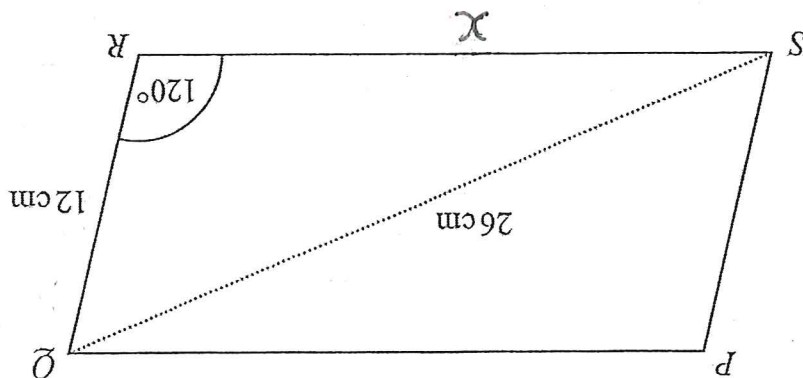
B : G

G : 1

1200 : 200

so we have $\frac{1200}{200}$ blue counters

57%



Calculate the area of the parallelogram PQRS.

Give your answer correct to 3 significant figures.

You must show all your working.

cosine rule to find side SR

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$26^2 = x^2 + 12^2 - 2 \times x \times 12 \cos 120$$

$$676 = x^2 + 144 + 12x$$

$$0 = x^2 + 12x - 532$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \times -532}}{2} = 17.832 \dots$$

$$\text{Area PQRS} = 2 \times \frac{1}{2} \times 17.832 \dots \times 12 \sin 120$$

$$= 185.323 \dots$$

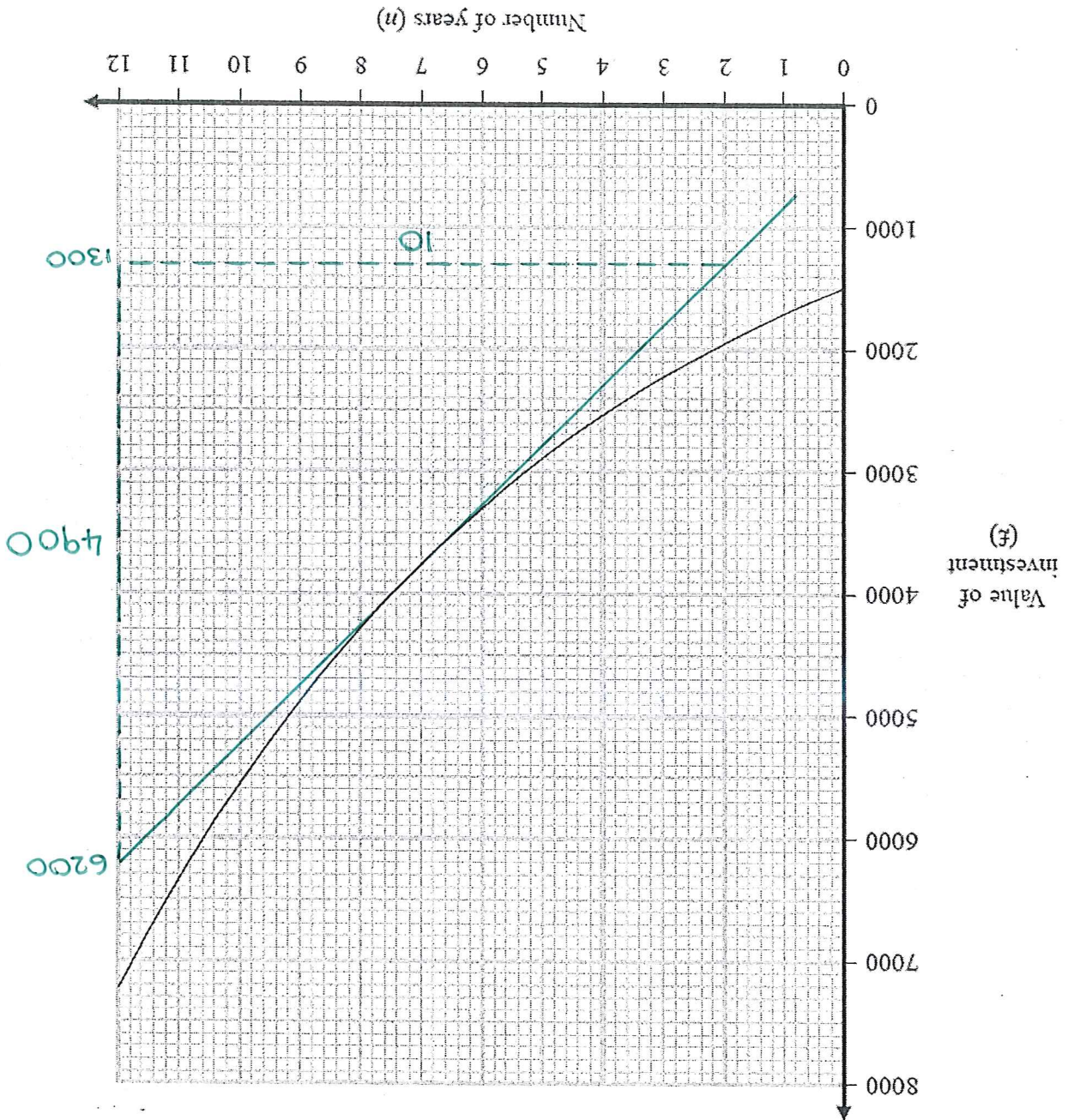
$$= 185 \text{ cm}^2 \text{ (3 s.f.)}$$

.....cm²

(Total for Question 17 is 5 marks)

18 Frank invested an amount of money for 12 years.

The graph shows the value of Frank's investment over the 12 year period.



(a) (i) Write down the amount of money that Frank invested.

£1500

31 000

(Total for Question 18 is 6 marks)

(1)

When $n=7$, the value of the investment is increasing by £490 per year.

(ii) Explain what this gradient represents.

(3)

490

M1 suitable tangent at $n=7$
M1 full method to find gradient at $n=7$
A1 answer in range 450 to 500 (or ft from "tangent")

$$\frac{4900}{10} = 490$$

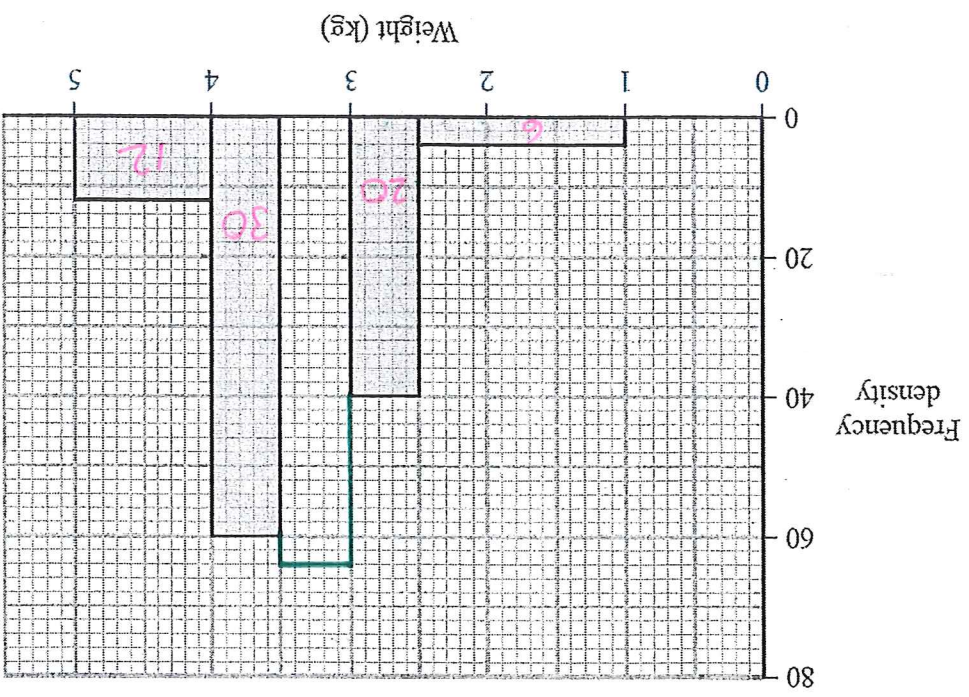
(b) (i) Find an estimate for the gradient of the curve at $n=7$

(1)

7200 = 4.8 He was not correct. It was worth 1500
4.8 times its original value.

(ii) Was Frank correct?
Give a reason for your answer.

Frank said that 12 years after he had made his investment, it was worth more than five times its original value.



The incomplete histogram shows information about the weights of 100 babies.
All 100 babies have a weight between 1 kg and 5 kg.

6 of the babies have a weight between 1 kg and 2.5 kg.

Complete the histogram.
You must show all your working.

$$\begin{aligned}
 &1-2.5 \text{ kg} : 6 \\
 &2.5-3 \text{ kg} : 0.5 \times 40 = 20 \\
 &3-3.5 \text{ kg} : ? \\
 &3.5-4 \text{ kg} : 0.5 \times 60 = 30 \\
 &4-5 \text{ kg} : 1 \times 12 = 12 \\
 &32 \text{ babies } 3-3.5 \text{ kg} \\
 &32 \div 0.5 = 64
 \end{aligned}$$

$$6 + 20 + 30 + 12 = 68$$

m1 any correct frequency
m1 find all correct frequencies (20, 30, 12)
m1 finds missing frequency 32
C1 bar correctly drawn

(Total for Question 19 is 4 marks)

Two bags, A and B, each contain only green marbles and red marbles.

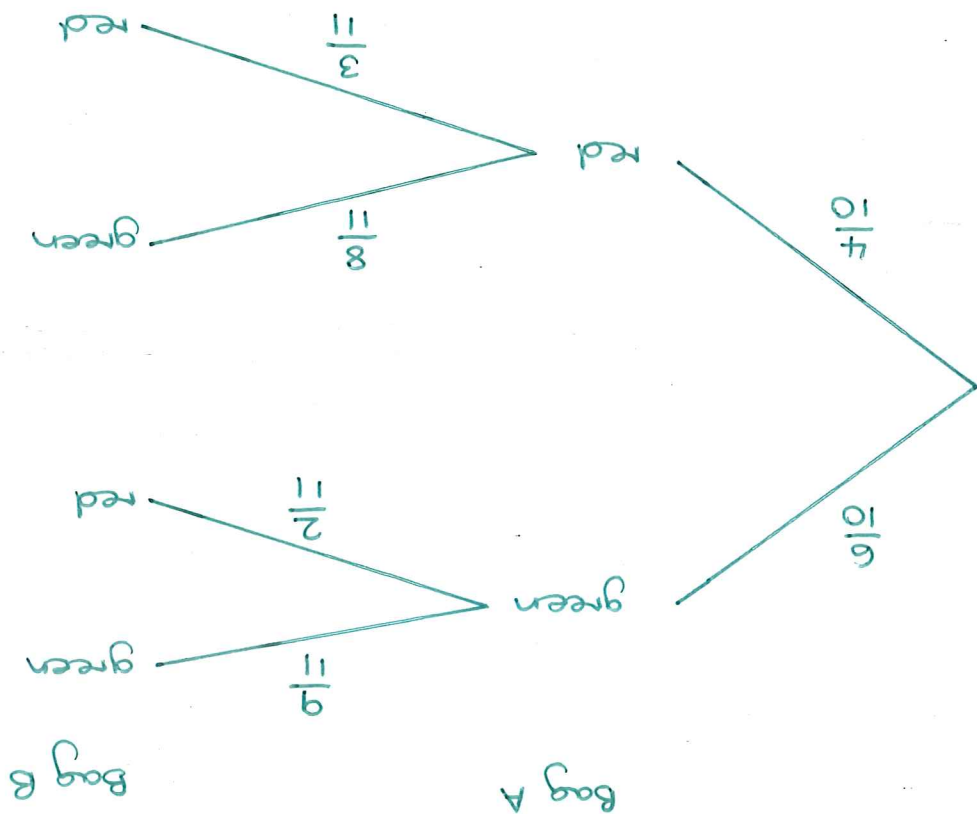
There are 6 green marbles and 4 red marbles in bag A.

There are 8 green marbles and 2 red marbles in bag B.

One marble is going to be taken at random from bag A and placed in bag B.

A marble is then going to be taken at random from bag B.

Work out the probability that this marble will be a green marble.



$$P(\text{green}) = \frac{27}{55} + \frac{16}{55} = \frac{43}{55}$$

P1 either $\frac{9}{11}$ or $\frac{8}{11}$

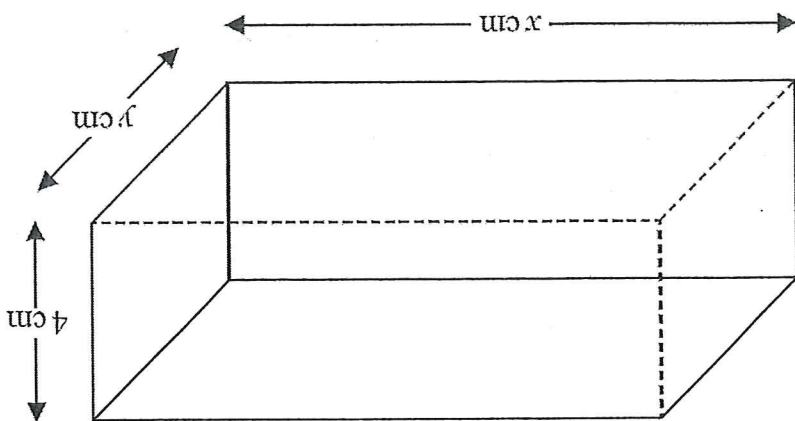
P1 any product from tree

P1 complete process

(Total for Question 20 is 4 marks)

0.781

At 0.781



The volume of the cuboid is 119 cm^3
The total surface area of the cuboid is 155.5 cm^2

Given that $x > y$, work out the value of x and the value of y .
You must show all your working.

Volume = 119

$$4xy = 119$$

M1

$$y = \frac{119}{4x}$$

Surface area = 155.5

$$2 \times 4y + 2 \times xy + 2 \times 4x = 155.5$$

$$8y + 2xy + 8x = 155.5$$

M1

Substitute: $8 \times \frac{119}{4x} + 2x \times \frac{119}{4x} + 8x = 155.5$

M1 Find equation in one variable

$$\frac{238}{x} + \frac{119}{2} + 8x = 155.5$$

$$4x^2 - 48x + 119 = 0$$

$$16x^2 - 192x + 476 = 0$$

$$-311x \quad \times 2x \quad 4x^2 - 48x + 119 = 0$$

$$(2x - 7)(2x - 17) = 0$$

$$x = \frac{7}{2} \text{ or } x = \frac{17}{2}$$

$$y = \frac{17}{2} \quad y = \frac{7}{2}$$

(Total for Question 21 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS