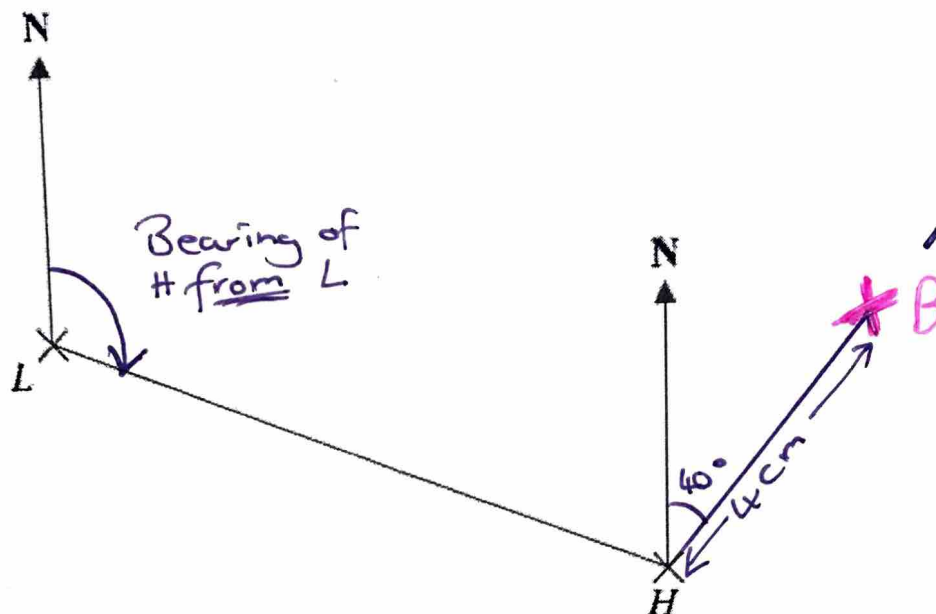


Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. The diagram shows the position of a lighthouse L and a harbour H .



The scale of the diagram is 1 cm represents 5 km.

- (a) Work out the real distance between L and H .

$$8.5 \text{ cm} \\ \times 5$$

$$\dots\dots\dots 42.5 \text{ km} \\ (1)$$

- (b) Measure the bearing of H from L .

$$\dots\dots\dots 110^\circ \\ (1)$$

A boat B is 20 km from H on a bearing of 040° .

- (c) On the diagram, mark the position of boat B with a cross ($\times$).
Label it B .

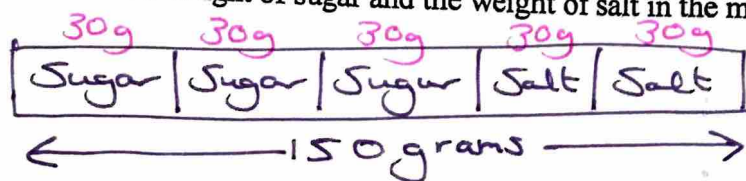
$$20 \text{ km} \div 5 = 4 \text{ cm}$$

(2)

(Total for Question 1 is 4 marks)

2. A mixture of sugar and salt is in the ratio 3 : 2
The weight of the mixture is 150 grams.

(a) Calculate the weight of sugar and the weight of salt in the mixture.



$$150 \div 5 = 30$$

$$\begin{aligned}\text{Sugar} &= 3 \times 30\text{g} \\ &= 90\text{g}\end{aligned}$$

$$\begin{aligned}\text{Salt} &= 2 \times 30\text{g} \\ &= 60\text{g}\end{aligned}$$

Sugar 90 g
Salt 60 g
(3)

30 grams of sugar and 10 grams of salt are added to the mixture.

(b) Calculate the ratio of sugar to salt in the new mixture.

$$\text{Sugar } 90\text{g} + 30\text{g} = 120\text{g}$$

$$\text{Salt } 60\text{g} + 10\text{g} = 70\text{g}$$

Sugar : Salt

$$\begin{array}{ccc} 120 & : & 70 \\ \div 10 \downarrow & & \downarrow \div 10 \\ 12 & : & 7 \end{array}$$

..... 12 : 7
(2)

(Total for Question 2 is 5 marks)

sugar : salt

$$\begin{array}{ccc} 12 & : & 7 \\ \div 7 \downarrow & & \downarrow \div 7 \\ 1.71 & : & 1 \end{array}$$

Mark scheme aster for this format but wording in question did not make this clear - remember these are practice papers and have not been through full checks.

3. $A = 2^2 \times 3 \times 5^2$

$B = 2^3 \times 5$

(a) Find the Highest Common Factor (HCF) of A and B .

$A = 2 \times 2 \times 3 \times 5 \times 5$

$B = 2 \times 2 \times 2 \times 5$

$HCF = 2 \times 2 \times 5$
 $= 20$

20

(1)

(b) Find the Lowest Common Multiple (LCM) of A and B .

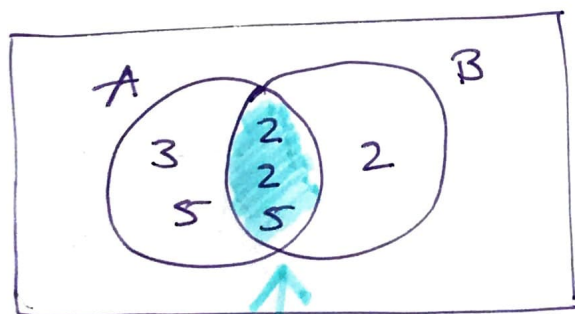
$LCM = 20 \times 3 \times 5 \times 2$
 $= 600$

600

(2)

(Total for Question 3 is 3 marks)

Venn Diagram Method



HCF
 $= 2 \times 2 \times 5$
 $= 20$

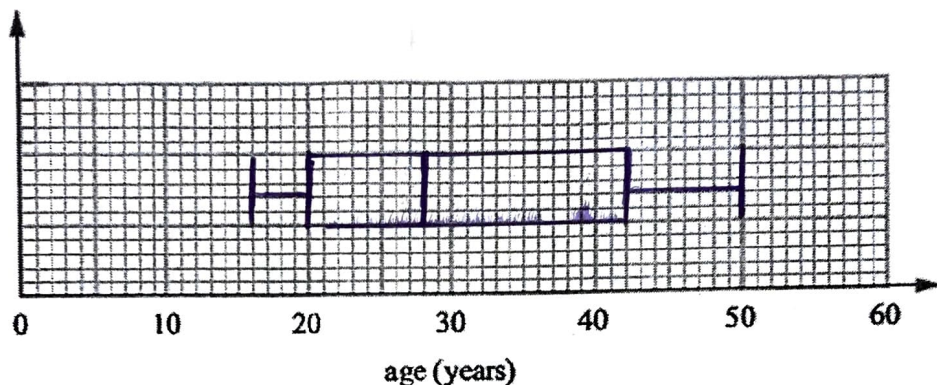
$LCM = 3 \times 5 \times 20 \times 2$
 $= 600$

4. Here are the ages, in years, of 15 women at West Ribble Tennis Club.

Smallest → 16, 18, 18, 20, 25, 25, 27, 28, 30, 35, 38, 42, 45, 46, 50 → Biggest

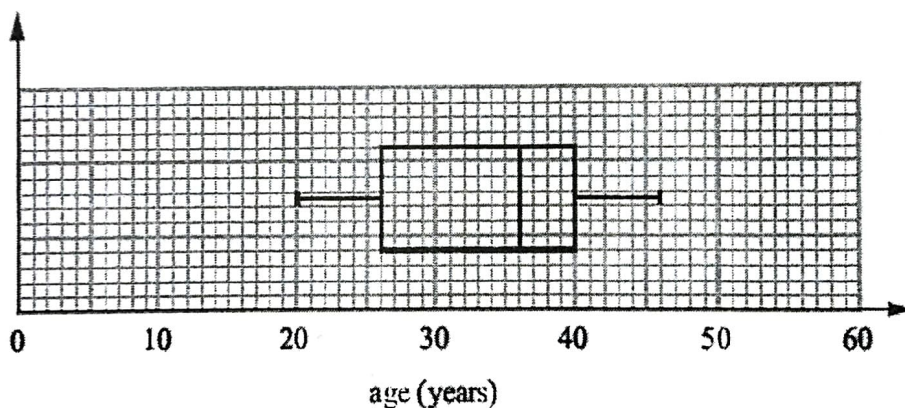
Handwritten notes: LQ (circled 20), median (circled 28), UQ (circled 42)

(a) On the grid, draw a box plot for this information.



(3)

The box plot below shows the distribution of the ages of the men at West Ribble Tennis Club.



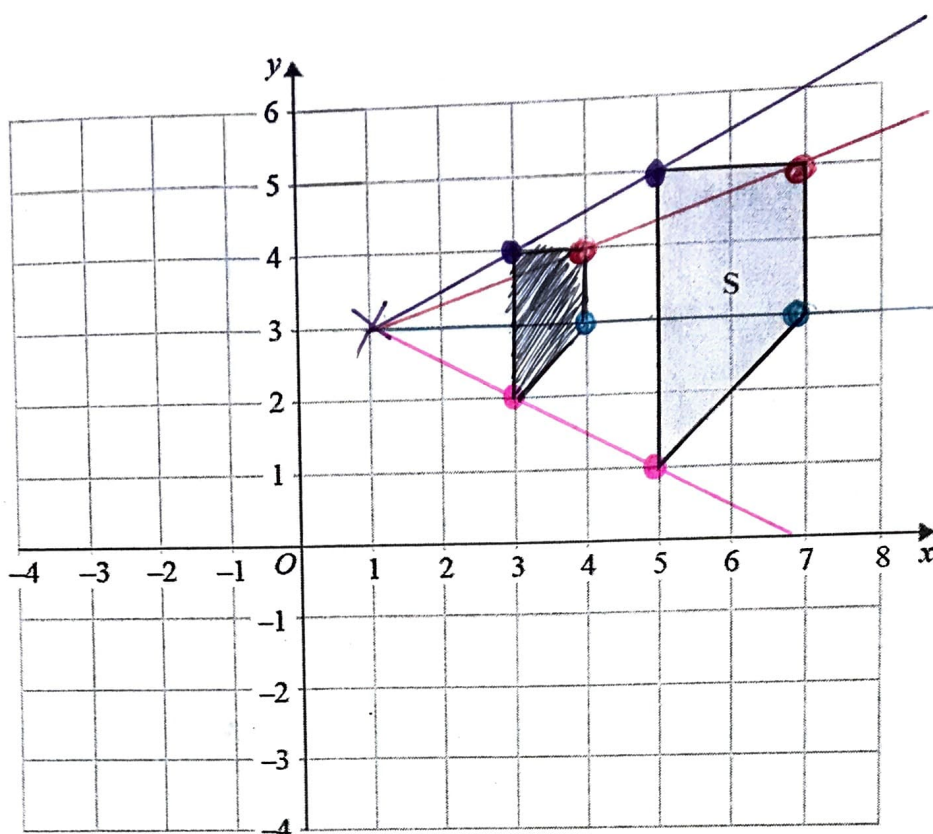
(b) Use the box plots to compare the distributions of the ages of these women and the distributions of the ages of these men.

compare medians
compare IQR
use context. (2)

(Total for Question 4 is 5 marks)

- The Median age of men at West Ribble Tennis Club is 36 compared with the median age of women at 28. This shows that on average the men at West Ribble Tennis Club are older than the women.
- The Interquartile range of the ages of women is larger than the IQR of the ages of men. This shows that there is a greater variation of ages of women at West Ribble Tennis Club than men.

5.



Enlarge shape S with scale factor $\frac{1}{2}$ and centre (1, 3).

(Total for Question 5 is 2 marks)

6. Given that, for all values of x ,

$$6x^3 + 7x^2 - 56x + 48 = (2x^2 + kx - 12)(3x - 4), \text{ where } k \text{ is a constant,}$$

find the value of k .

$$6x^3 + 7x^2 - 56x + 48 \equiv 6x^3 - 8x^2 + 3kx^2 - 4kx - 36x + 48$$

$$6x^3 + 7x^2 - 56x + 48 \equiv 6x^3 + (3k - 8)x^2 + (-4k - 36)x + 48$$

comparing coefficients of x^2

$$\begin{array}{rcl} 7 & = & 3k - 8 \quad [+8] \\ 15 & = & 3k \quad [\div 3] \\ 5 & = & k \end{array}$$

or we could have compared coefficients of x

$$\begin{array}{rcl} -56 & = & -4k - 36 \quad [+36] \\ -20 & = & -4k \quad [\div -4] \\ 5 & = & k \end{array}$$

$$k = 5$$

(Total for Question 6 is 2 marks)

7.

Shaded Area
 $= \text{Area Circle} - \text{Area Square}$

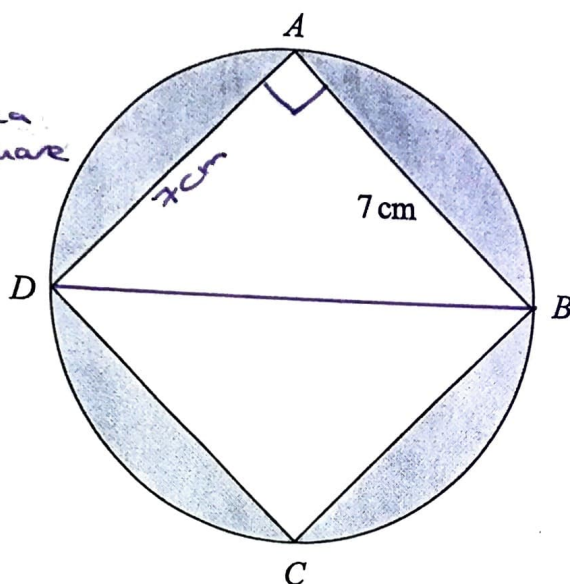


Diagram NOT
 accurately drawn

$$\begin{aligned}\text{Area Square} &= 7 \times 7 \\ &= 49 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area Circle} &= \pi r^2\end{aligned}$$

need to find the
 radius r

A, B, C and D are points on a circle.
 $ABCD$ is a square of side 7 cm.

Work out the total area of the shaded regions.
 Give your answer correct to the nearest whole number.

DB is the diameter of the circle
 find DB using Pythagoras' theorem

$$\begin{aligned}DB^2 &= 7^2 + 7^2 \\ &= 98\end{aligned}$$

$$\begin{aligned}DB &= \sqrt{98} \\ &= 7\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Diameter} &= 7\sqrt{2} \\ \text{radius} &= \frac{7\sqrt{2}}{2}\end{aligned}$$

$$\begin{aligned}\text{Area Circle} &= \pi \times \left(\frac{7\sqrt{2}}{2}\right)^2 \\ &= \frac{49}{2}\pi\end{aligned}$$

$$\text{Shaded Area} = \frac{49}{2}\pi - 49$$

$$\begin{aligned}&= 27.9690 \\ &= 28 \text{ cm}^2\end{aligned}$$

..... 28 cm^2

(Total for Question 7 is 5 marks)

(to nearest whole number)

8. Danielle invested £2800 for n years in a savings account.
 She was paid 2.5% per annum compound interest.
 The interest is paid into the account at the end of each year.
 At the end of n years, the amount of money in the savings account is greater than £3000 for the first time.

Work out the value of n .

$$100\% + 2.5\% = 102.5\% = 1.025$$

$$£2800 \times (1.025)^n > £3000 \quad n = ?$$

$$2800 \times 1.025^n > 3000$$

use trial and improvement to find n

try $n=3$ $2800 \times 1.025^3 = £3015.29375$
 $n=2$ $2800 \times 1.025^2 = £2941.75$

after 2 years Danielle has £2941.75

after 3 years Danielle has £3015.29

..... $n=3$

(Total for Question 8 is 3 marks)

9. n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n-2)^2$ is always an even number.

$$\begin{aligned} & n^2 - 2 - [(n-2)(n-2)] \\ &= n^2 - 2 - [n^2 - 2n - 2n + 4] \\ &= n^2 - 2 - 1[n^2 - 4n + 4] \\ &= \cancel{n^2} - 2 - \cancel{n^2} + 4n - 4 \\ &= -6 + 4n \\ &= 4n - 6 \\ &= 2(2n - 3) \end{aligned}$$

2 is a factor and therefore the expression is an even number.

(Total for Question 9 is 2 marks)

10. Make e the subject of $k = \sqrt{\frac{5m+2e}{3e}}$ [square

$$k^2 = \frac{5m+2e}{3e} \quad [\times 3e]$$

$$3ek^2 = 5m + 2e \quad [-2e]$$

$$3ek^2 - 2e = 5m \quad [\text{factorise}]$$

$$e(3k^2 - 2) = 5m \quad [\div (3k^2 - 2)]$$

$$e = \frac{5m}{3k^2 - 2}$$

(Total for Question 10 is 4 marks)

11. (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

$$x^3 + 4x - 1 = 0$$

$$x = 0 \quad (0)^3 + 4(0) - 1 = -1$$

$$x = 1 \quad (1)^3 + 4(1) - 1 = 4$$

change of sign therefore solution lies between $x = 0$ and $x = 1$ (2)

- (b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

notice that this x has been made the subject.

$$\begin{aligned} x^3 + 4x &= 1 & [-x^3] \\ 4x &= 1 - x^3 & [\div 4] \\ x &= \frac{1 - x^3}{4} & \text{or } x = \frac{1}{4} - \frac{x^3}{4} \end{aligned}$$

(1)

- (c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

$$x_0 = 0$$

$$x_1 = \frac{1}{4} - \frac{x_0^3}{4} = \frac{1}{4} - \frac{0^3}{4} = 0.25$$

$$x_2 = \frac{1}{4} - \frac{x_1^3}{4} = \frac{1}{4} - \frac{0.25^3}{4} = 0.24609375$$

or use ANS button on calculator.

$$0 =$$

$$\frac{1}{4} - \frac{\text{ANS}^3}{4}$$

$$=$$

$$=$$

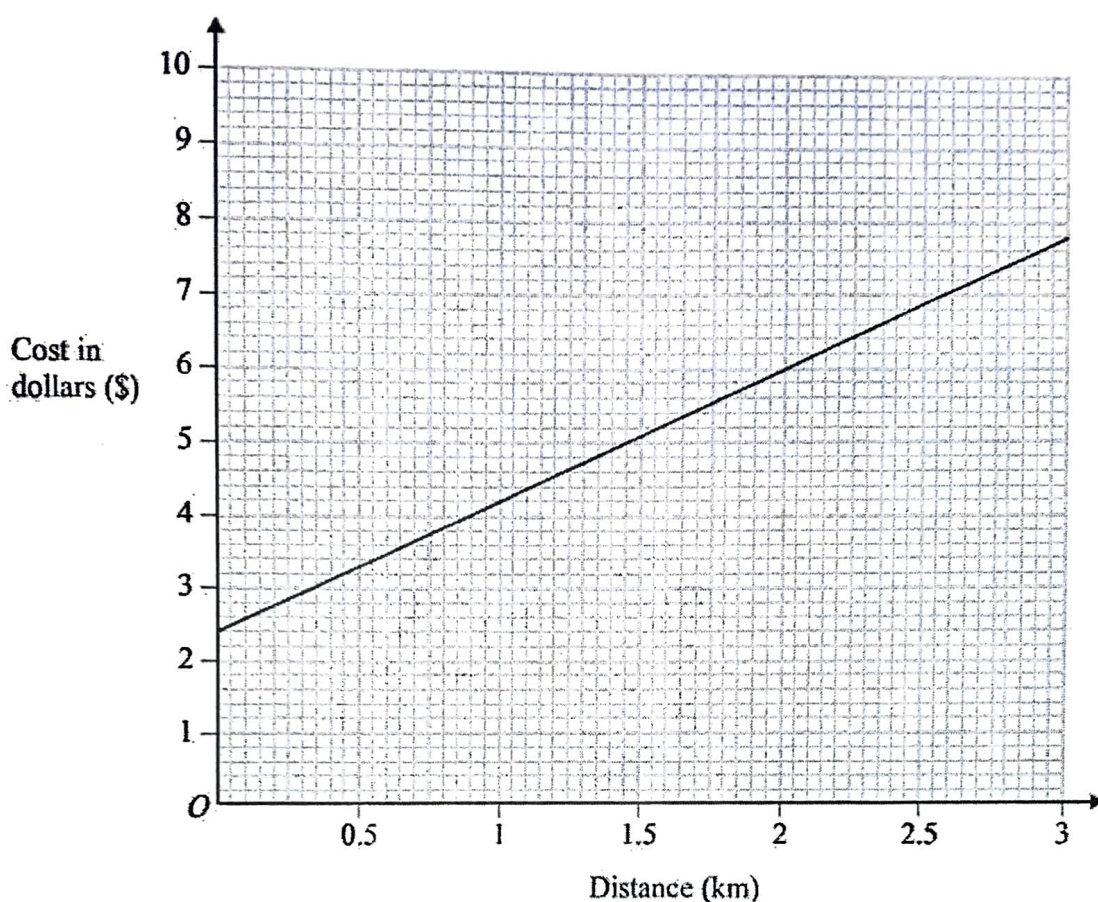
x_1 and x_2 are equal correct to 2sf

$$\dots\dots\dots 0.25$$

(3)

(Total for Question 11 is 6 marks)

12.



The graph gives information about the costs of taxi journeys of different distances. The cost of a taxi journey consists of a **fixed initial charge** and a **charge per km**.

- (a) Give an interpretation of the intercept of the graph on the y-axis.

when 0 km is travelled the cost is \$2.40

the fixed initial charge
(1)

- (b) Give an interpretation of the gradient of the graph.

the amount the cost goes up (RISE) for every km travelled (RUN)

the charge per km
(1)

(Total for Question 12 is 2 marks)

13. $f(x) = \frac{4}{x-3}$ $g(x) = \frac{x-2}{x}$

(a) Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

let $y = \frac{4}{x-3}$

Swap x and y

$x = \frac{4}{y-3}$

rearrange to make y the subject

$x(y-3) = 4$

$xy - 3x = 4$

$xy = 4 + 3x$

$y = \frac{4+3x}{x}$

$f^{-1}(x) = \frac{4+3x}{x}$ (3)

(b) Solve $fg(a) = 1$

You must show your working.

$g(x) = \frac{x-2}{x}$

$g(a) = \frac{a-2}{a}$

$f[g(a)] = f\left[\frac{a-2}{a}\right]$

$= \frac{4}{\frac{a-2}{a} - 3}$

Solve $fg(a) = 1$

$\frac{4}{\frac{a-2}{a} - 3} = 1$ [$\times \frac{a-2}{a} - 3$]

$4 = \frac{a-2}{a} - 3$ [$+3$]

$7 = \frac{a-2}{a}$ [$\times a$]

$7a = a - 2$ [$-a$]

$6a = -2$ [$\div 6$]

$a = -\frac{2}{6}$

$a = -\frac{1}{3}$

$a = -\frac{1}{3}$ (3)

(Total for Question 13 is 6 marks)

14. Here is a solid bar made of metal.

The bar is in the shape of a cuboid.

The height of the bar is h cm.

The base of the bar is a square of side d cm.

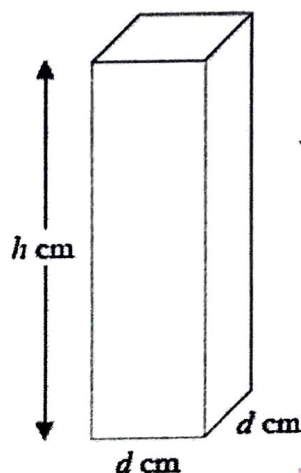
The mass of the bar is M kg.

$d = 8.3$ correct to 1 decimal place.

$M = 13.91$ correct to 2 decimal places.

$h = 84$ correct to the nearest whole number.

use upper and lower bounds



$$\text{Volume} = hd^2$$

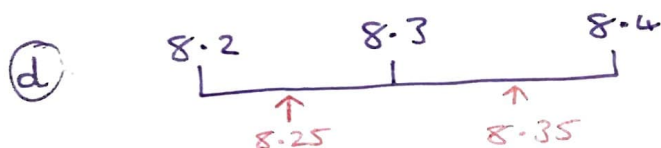
Find the value of the density of the metal to an appropriate degree of accuracy.

Give your answer in g/cm^3 .

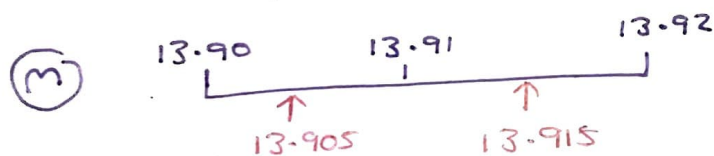
find lower and upper bound value of density.



You must explain why your answer is to an appropriate degree of accuracy.



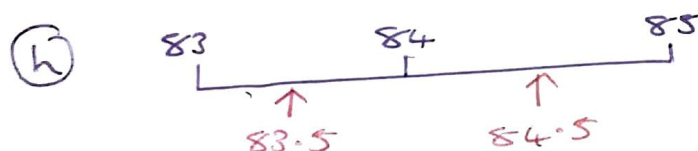
$$8.25 \leq d < 8.35$$



$$13.905 \leq m < 13.915$$

convert to grams

$$13905 \leq m < 13915$$



$$83.5 \leq h < 84.5$$

$$\text{Density} = \frac{\text{mass}}{\text{Volume}}$$

$$\begin{aligned} \text{Lower bound Density} &= \frac{\text{lower bound mass}}{\text{upper bound Volume}} \\ &= \frac{13905 \text{ grams}}{5891.55125 \text{ cm}^3} \\ &= 2.360159... \text{ g/cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Upper bound Density} &= \frac{\text{upper bound mass}}{\text{lower bound Volume}} \\ &= \frac{13915 \text{ grams}}{5683.21875 \text{ cm}^3} \\ &= 2.44843... \text{ g/cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Lower bound Volume} &= \text{lower bound } h \times (\text{lower bound } d)^2 \\ &= 83.5 \times (8.25)^2 \\ &= 5683.21875 \end{aligned}$$

$$\begin{aligned} \text{Upper bound Volume} &= \text{upper bound } h \times (\text{upper bound } d)^2 \\ &= 84.5 \times (8.35)^2 \\ &= 5891.55125 \end{aligned}$$

(Total for Question 14 is 5 marks)

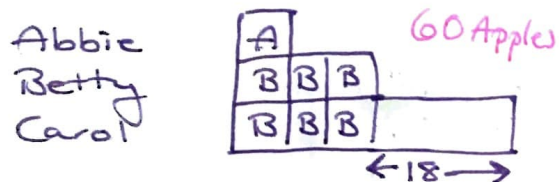
Lower bound and upper bound density both round to 2.4 g/cm^3 correct to 1dp. Therefore density = 2.4 g/cm^3 (1dp).

$$A : B : C$$

15. 60 apples are shared between Abbie, Betty and Carol in the ratios $1 : 3 : x$, where $x > 3$.

The number of apples in Carol's share is 18 more than the number of apples in Betty's share.

Find the value of x .



Abbie has 6 apples
Betty has 18 apples
Carol has $18 + 18 = 36$ apples

$$A : B : C$$

$$6 : 18 : 36$$

$$\div 6 \quad \div 6 \quad \div 6$$

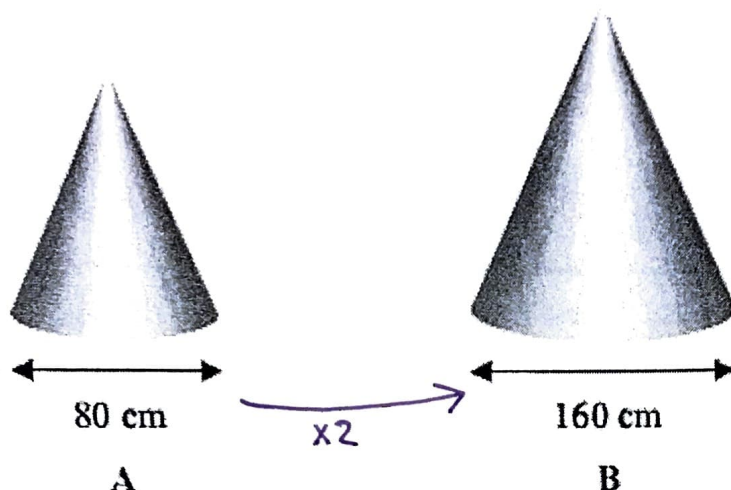
$$1 : 3 : 6$$

$$x = 6$$

(Total for Question 15 is 4 marks)

16. Ali has two solid cones made from the same type of metal.

Diagram NOT accurately drawn



The two solid cones are mathematically similar.

The base of cone A is a circle with diameter 80 cm.

The base of cone B is a circle with diameter 160 cm.

Ali uses 80 ml of paint to paint cone A.

Ali is going to paint cone B.

Ali will be painting the Surface Area of Cone B

- (a) Work out how much paint, in ml, he will need.

Scale Factor of length $k = 2$
 Scale Factor of area $k^2 = 2^2 = 4$

To paint the surface area of cone B
 Ali will need

$$80 \text{ ml} \times 4 = \dots\dots\dots 320 \text{ ml}$$

(2)

The volume of cone A is $171\,700 \text{ cm}^3$.

- (b) Work out the volume of cone B.

Scale factor of volume $k^3 = 2^3 = 8$

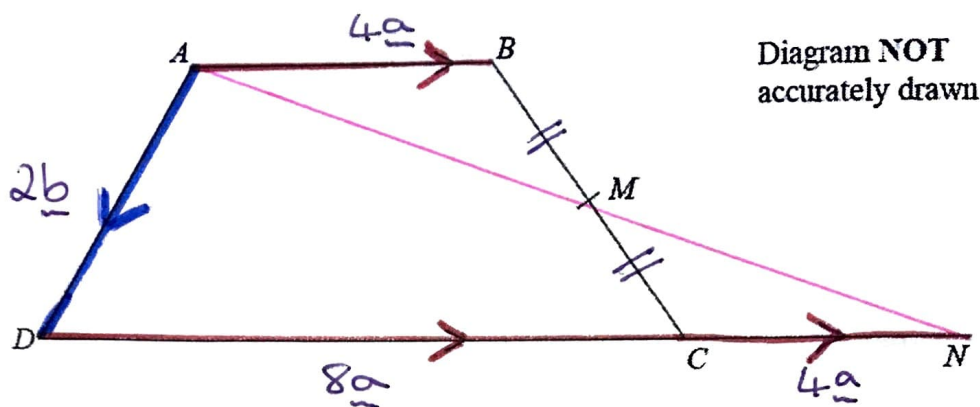
$$\begin{aligned} \text{Volume of cone B} &= \text{Volume of cone A} \times 8 \\ &= 171\,700 \times 8 \end{aligned}$$

$$= \dots\dots\dots 1\,373\,600 \text{ cm}^3$$

(3)

(Total for Question 16 is 5 marks)

17.



AB is parallel to DC

$$DC = 2AB$$

M is the midpoint of BC

$$\vec{AD} = 2\mathbf{b}$$

$$\vec{AB} = 4\mathbf{a}$$

(a) Find $\vec{BM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Give your answer in its simplest form.

$$\begin{aligned}\vec{BC} &= \vec{BA} + \vec{AD} + \vec{DC} \\ &= -4\mathbf{a} + 2\mathbf{b} + 8\mathbf{a} \\ &= 4\mathbf{a} + 2\mathbf{b}\end{aligned}$$

$$\begin{aligned}\vec{BM} &= \frac{1}{2} \vec{BC} \\ &= \frac{1}{2} (4\mathbf{a} + 2\mathbf{b}) \\ &= 2\mathbf{a} + \mathbf{b}\end{aligned}$$

(2)

N is the point such that DCN is a straight line and $DC : CN = 2 : 1$

(b) Show that AMN is a straight line.

$$\begin{array}{ccc} DC : CN & & \\ 2 : 1 & & \\ \times 4 \downarrow & & \downarrow \times 4 \\ 8\mathbf{a} : 4\mathbf{a} & & \end{array}$$

$$\vec{BM} = \vec{MC}$$

$$\begin{aligned}\vec{AM} &= \vec{AB} + \vec{BM} \\ &= 4\mathbf{a} + 2\mathbf{a} + \mathbf{b} \\ &= 6\mathbf{a} + \mathbf{b}\end{aligned}$$

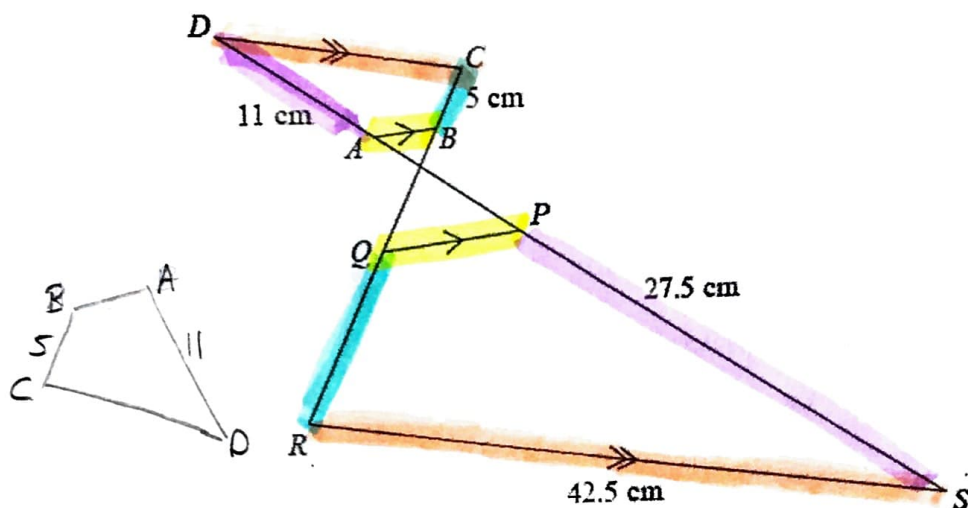
$$\begin{aligned}\vec{AN} &= \vec{AD} + \vec{DC} + \vec{CN} \\ &= 2\mathbf{b} + 8\mathbf{a} + 4\mathbf{a} \\ &= 12\mathbf{a} + 2\mathbf{b}\end{aligned}$$

$\vec{AN} = 2\vec{AM}$ and therefore $\vec{AN}$ and $\vec{AM}$ are parallel and both pass through a common point N (2)

(Total for Question 17 is 4 marks)

Since $\vec{AM}$ and $\vec{AN}$ are parallel and pass through N we can conclude that AMN is a straight line

18. In the diagram, $DAPS$ and $CBQR$ are straight lines.
 AB is parallel to QP and DC is parallel to RS .
 $AD = 11$ cm, $BC = 5$ cm, $PS = 27.5$ cm and $RS = 42.5$ cm.



Quadrilateral $ABCD$ is similar to quadrilateral $PQRS$.

- (a) Work out the length of RQ .

we can use the corresponding sides AD and PS to find the scale factor

$$27.5 \div 11 = 2.5$$

$$\text{Scale factor} = 2.5$$

RQ corresponds to side BC

$$\text{length of } RQ = 5 \times 2.5 = 12.5$$

cm

(2)

- (b) Work out the length of CD .

CD corresponds to side RS

$$\text{length of } CD = 42.5 \div 2.5 = 17$$

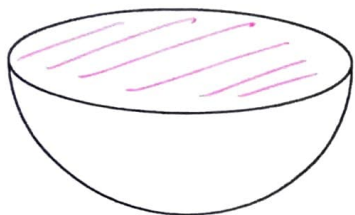
17

cm

(2)

(Total for Question 18 is 4 marks)

19. The diagram shows a solid hemisphere.



Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

The hemisphere has a **total** surface area of $\frac{16}{3}\pi \text{ cm}^2$

The hemisphere has a volume of $k\pi \text{ cm}^3$

Find the value of k .

Surface Area of hemisphere

$$\begin{aligned}
 &= \frac{1}{2} \text{ surface Area of Sphere} + \text{area circle} \\
 &= \frac{1}{2} \times 4\pi r^2 + \pi r^2 \\
 &= 2\pi r^2 + \pi r^2 \\
 &= 3\pi r^2
 \end{aligned}$$

the shaded surface on picture

therefore $3\pi r^2 = \frac{16}{3}\pi$

$[\div \pi]$

$$3r^2 = \frac{16}{3}$$

$[\div 3]$

$$r^2 = \frac{16}{9}$$

$[\sqrt{\quad}]$

$$r = \frac{4}{3}$$

the radius r cannot be negative so we don't need plus or minus

Volume of hemisphere

$$\begin{aligned}
 &= \frac{1}{2} \text{ Volume of Sphere} \\
 &= \frac{1}{2} \times \frac{4}{3}\pi r^3 \\
 &= \frac{4}{6}\pi r^3
 \end{aligned}$$

we know that $r = \frac{4}{3}$

so volume

$$\begin{aligned}
 &= \frac{4}{6} \times \pi \times \left(\frac{4}{3}\right)^3 \\
 &= \frac{128}{81}\pi
 \end{aligned}$$

the Volume = $k\pi \text{ cm}^3$

$$\therefore k = \frac{128}{81}$$

..... or 1.58 (3sf)

(Total for Question 19 is 4 marks)

20. ABC is a triangle.

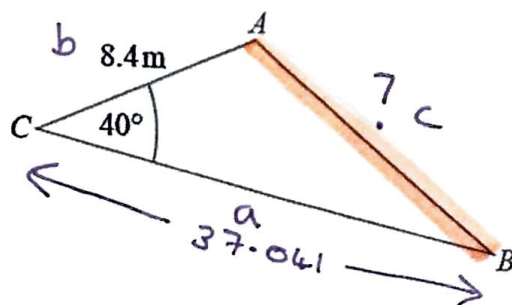


Diagram NOT
accurately drawn

$AC = 8.4$ m
Angle $ACB = 40^\circ$

The area of the triangle = 100 m^2 .

Work out the length of AB .

Give your answer correct to 3 significant figures.

You must show all your working.

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

$$100 = \frac{1}{2} \times a \times 8.4 \times \sin 40$$

$$100 = 2.699707961 a$$

$$37.041 = a$$

$$\text{length of } BC = 37.0410435$$

Now we can use the "easy" cosine rule to find the length of side AB

40° is easy between two known sides!

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= (37.0410435)^2 + (8.4)^2 - 2 \times 8.4 \times 37.0410435 \times \cos 40$$

$$= 965.8974663$$

$$c = 31.07889101$$

$$= 31.1 \text{ m (3sf)}$$

31.1

..... m

(Total for Question 20 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS