**GCSE Mathematics (1MA1) – Higher Tier**

**Themed papers – Algebraic proof**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

**NOTES ON MARKING PRINCIPLES**

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| **Guidance on the use of codes within this mark scheme** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  P1 – process mark. This mark is generally given for setting up an appropriate process to find a solution in the context of the question.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  C1 – communication mark. This mark is given for explaining your answer or giving a conclusion in context supported by your working.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 3 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | (*x* + 3) × (*x* + 3) | M1 | This mark is given for writing the area using algebraic terms |
| *x*2+ 3*x* + 3*x* + 9 = 10 | M1 | This mark is given for expanding (*x* + 3)(*x* + 3) |
| *x*2 + 6*x* = 1 | A1 | This mark is given for rearranging to give the given expression |

**Question 2 (Total 4 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | Let an odd number be 2*n* + 1 | B1 | This mark is given for finding an algebraic representation of an odd number |
| (2*n* + 1)2 | M1 | This mark is given for an expression for the square of an odd number |
| 4*n*2 + 4*n* + 1 | A1 | This mark is given for expanding brackets |
| Thus 4(*n*2 + *n*) + 1 is 1 greater than a multiple of 4 | C1 | This mark is given for concluding statement |

**Question 3 (Total 4 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | (*n* – 2)2 = *n*2 – 4*n* + 4 | C1 | This mark is given for a correct expansion of (*n* – 2)2 |
| *n*2 – 2 − *n*2 + 4*n* − 4 | C1 | This mark is given for a correct expansion of *n* – 2 –(*n* – 2)2 |
| 2(2*n* − 3) | C1 | This mark is given for reducing the expression to (2*n* − 3) |
| 2(2*n*−3) always even since it has a factor of 2 for all values of *n* | C1 | This mark is given for a correct conclusion supported by working shown |

**Question 4 (Total 2 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working an or answer examiner might expect to see** | **Mark** | **Notes** |
|  | For even numbers 2*n*:  (2*n*)2 – 2*n* = 4*n*2 – 2*n* = 2(2*n*2 – *n*) so even  For odd numbers 2*n* + 1:  (2*n* + 1)2 – 2*n* + 1 = 4*n*2 + 4*n* + 1 – (2*n* + 1)  = 4*n*2 + 2*n*  = 2(*n*2 – *n*) so even  Thus for all integer value of *n*, *n*2 – *n* is never an odd number | C2 | This mark is given for a fully correct proof  (C1 is given for a partial explanation) |

**Question 5 (Total 3 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
|  | Let two consecutive even numbers be 2*n* and 2*n* + 2 | P1 | This mark is given for a method to algebraically represent two consecutive even numbers |
| (2*n*)2 + (2*n* + 2)2  = 4*n*2 + 4*n*2 + 8*n* + 4 | P1 | This mark is given for a process of  expanding both expressions with at least one expansion fully correct eg 4*n*2  and 4*n*2 + 4*n* + 4*n* +4 |
| = 4(2*n*2 + *n* + 1)  which is a multiple of 4 for any *n* | A1 | This mark is given for a complete and correct proof |

**Question 6 (Total 3 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | 2*n* – 1, 2*n* + 1 | C1 | This mark is given for a correct representation of two consecutive odd numbers |
| (2*n* + 1)2 – (2*n* – 1)2  = (4*n*2 + 4*n* + 1) – (4*n*2 – 4*n* + 1)  = 4*n*2 + 4*n* + 1 – 4*n*2 + 4*n* – 1 | C1 | This mark is given for multiplying out brackets and collecting terms |
| = 8*n*  Always a multiple of 8 | C1 | This mark is given for stating a correct conclusion |

**Question 7 (Total 2 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
|  | (*n*2 + *n* + *n*2 + *n* + 2*n* + 2) | 1 | This mark is given for expanding the brackets |
|  | = (2*n*2 + 4*n* + 2) = *n*2 + 2*n* + 1  = (*n* + 1)2  which is square for all integer values of *n* | 1 | This mark is given for a complete proof with reference to (*n* + 1)2 being square for all integers *n* |

**Question 8 (Total 5 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | *x* = 10 + 2*y*  (10 + 2*y*)2 + *y*2 = 20 | 1 | This mark is given for a method to start the process by finding a value for *x* in terms of *y* and substituting |
| (100 + 20*y* + 20*y* + 4*y*2) + *y*2 = 20  4*y*2 + 20*y* + 20*y* = 100 | 1 | This mark is given for expanding brackets on the expression obtained |
| 5*y*2 + 40*y* + 80 = 0 | 1 | This mark is given for forming a quadratic equation to be solved |
| (5*y* + 20) (*y* + 4) = 0  *y* = –4, *x* = 2 | 1 | This mark is given for solving the quadratic equation for *y* and so find the value of *x* |
| The line intersects the circle at only one point (2, –4), so must be a tangent | 1 | This mark is given for a fully correct statement to conclude the proof |

**Question 9 (Total 3 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | (*y* + *x*)= *k*(*y* − *x*) | 1 | This mark is given for setting up an equations from the information given |
| *ky* – *y* = *x* + *kx* | 1 | This mark is given for isolating x and y on opposite sides |
| *y*(*k* – 1) = *k*(*x* + 1) so *y* = | 1 | This mark is given for using correct algebra to reach a conclusion |

**Question 10 (Total 4 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | *N* = 100*a* + 10*b* + *c*  *K* = 100*c* + 10*b* + *a* | M1 | This mark is given for forming algebraic expressions for *N* and *K* |
| *N* – *K*  = 99*a* – 99*c* | M1 | This mark is given for finding the difference for their expressions |
| = 99(*a* – *c*), so a multiple of 99 | C1 | This mark is given for a concluding statement using the term 99(*a* – *c*) |
| (b) | Yes, it has no effect since the *b* terms cancel | C1 | This mark is given for a correct statement |

Performance data:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Q** | **Taken from** | | | **Total Marks available** | **TOPIC** | **Spec Ref** | **AO** | **% Mean marks** | **Edexcel mean averages Marks of candidates who achieved grade:** | | | | | | | | | | | | |
| **Q** | **Series** | **Paper** | **ALL** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **2** | | **1** | | **U** |
| **1** | 4 | June 2017 | 1H | 3 | Algebra | A4, G16 | 2 | 66 | 1.97 | 2.97 | 2.92 | 2.73 | 2.26 | 1.48 | 0.62 | 0.18 | - | - | | 0.08 | |
| **2** | 12 | June 2018 | 1H | 4 | Algebra | N6, A4, A6 | 2 | 36 | 1.43 | 3.82 | 3.28 | 2.33 | 1.20 | 0.41 | 0.13 | 0.04 | - | - | | 0.01 | |
| **3** | 16 | June 2017 | 1H | 4 | Algebra | A1, A4, A6 | 2 | 27 | 1.06 | 3.08 | 2.25 | 1.56 | 0.89 | 0.37 | 0.13 | 0.05 | - | - | | 0.02 | |
| **4** | 13 | June 2019 | 1H | 2 | Algebra | A6 | 2 | 17 | 0.34 | 1.29 | 0.72 | 0.41 | 0.23 | 0.10 | 0.03 | 0.01 | - | - | | 0.02 | |
| **5** | 15 | Nov 2019 | 3H | 3 | Algebra | A6 | 2 | 12 | 0.35 | 2.89 | 2.35 | 1.61 | 0.71 | 0.33 | 0.05 | 0.00 | - | - | | 0.01 | |
| **6** | 15 | Nov 2018 | 3H | 3 | Number | N10 | 1 | 11 | 0.34 | 2.90 | 2.47 | 1.45 | 0.93 | 0.37 | 0.07 | 0.02 | - | - | | 0.01 | |
| **7** | 17 | Nov 2017 | 1H | 2 | Algebra | A4, A6 | 2 | 6 | 0.12 | 1.75 | 1.35 | 0.71 | 0.36 | 0.21 | 0.07 | 0.03 |  |  | | 0.01 | |
| **8** | 19 | Nov 2017 | 3H | 5 | Algebra | A19, A16 | 1 | 2 | 0.12 | 4.62 | 1.88 | 1.08 | 0.34 | 0.19 | 0.03 | 0.01 |  |  | | 0.00 | |
| **9** | 14 | Nov 2017 | 1H | 3 | Ratio | R4, R8, A5 | 2 | 2 | 0.06 | 2.25 | 1.59 | 0.51 | 0.22 | 0.05 | 0.02 | 0.01 |  |  | | 0.00 | |
| **10a** | 17a | Mock Set 2 | 3H | 3 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | | 3 | |
| **10b** | 17b | Mock Set 2 | 3H | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | | 1 | |
|  |  |  |  | **33** |  |  |  |  | **5.79** | **25.57** | **18.81** | **12.39** | **7.14** | **3.51** | **1.15** | **0.35** |  |  | | **33** | |