**Solving quadratic equations by factorisation**

Key points

* A quadratic equation is an equation in the form *ax*2 + *bx* + *c* = 0 where *a* ≠ 0.
* To factorise a quadratic equation find two numbers whose sum is *b* and whose products is *ac*.
* When the product of two numbers is 0, then at least one of the numbers must be 0.
* If a quadratic can be solved it will have two solutions (these may be equal).

Examples

**Example 1** Solve 5*x*2 = 15*x*

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| --- | --- |
| 5*x*2 = 15*x*  5*x*2 − 15*x* = 0  5*x*(*x* − 3) = 0  So 5*x* = 0 or (*x* − 3) = 0  Therefore *x* = 0 or *x* = 3 | **1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero.  Do not divide both sides by *x* as this would lose the solution *x* = 0.  **2** Factorise the quadratic equation.  5*x* is a common factor.  **3** When two values multiply to make zero, at least one of the values must be zero.  **4** Solve these two equations. |

**Example 2** Solve *x*2 + 7*x* + 12 = 0

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| *x*2 + 7*x* + 12 = 0  *b* = 7, *ac* = 12  *x*2 + 4*x* + 3*x* + 12 = 0  *x*(*x* + 4) + 3(*x* + 4) = 0  (*x* + 4)(*x* + 3) = 0  So (*x* + 4)= 0 or (*x* + 3) = 0  Therefore *x* = −4 or *x* = −3 | **1** Factorise the quadratic equation. Work out the two factors of *ac* = 12 which add to give you *b* = 7.  (4 and 3)  **2** Rewrite the *b* term (7*x*) using these two factors.  **3** Factorise the first two terms and the last two terms.  **4** (*x* + 4) is a factor of both terms.  **5** When two values multiply to make zero, at least one of the values must be zero.  **6** Solve these two equations. |

**Example 3** Solve 9*x*2 − 16 = 0

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| 9*x*2 − 16 = 0  (3*x* + 4)(3*x* – 4) = 0  So (3*x* + 4) = 0 or (3*x* – 4) = 0  or | **1** Factorise the quadratic equation. This is the difference of two squares as the two terms are (3*x*)2 and (4)2.  **2** When two values multiply to make zero, at least one of the values must be zero.  **3** Solve these two equations. |

**Example 4** Solve 2*x*2 − 5*x* − 12 = 0

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| *b* = −5, *ac* = −24  So 2*x*2 − 8*x* + 3*x* – 12 = 0  2*x*(*x* − 4) + 3(*x* − 4) = 0  (*x* – 4)(2*x* + 3) = 0  So (*x* – 4) = 0 or (2*x* +3) = 0  or | **1** Factorise the quadratic equation. Work out the two factors of *ac* = −24 which add to give you *b* = −5.  (−8 and 3)  **2** Rewrite the *b* term (−5*x*) using these two factors.  **3** Factorise the first two terms and the last two terms.  **4** (*x* − 4) is a factor of both terms.  **5** When two values multiply to make zero, at least one of the values must be zero.  **6** Solve these two equations. |

Practice

**1** Solve

**a** 6*x*2 + 4*x* = 0 **b** 28*x*2 – 21*x* = 0

**c** *x*2 + 7*x* + 10 = 0 **d** *x*2 – 5*x* + 6 = 0

**e** *x*2 – 3*x* – 4 = 0 **f** *x*2 + 3*x* – 10 = 0

**g** *x*2 – 10*x* + 24 = 0 **h** *x*2 – 36 = 0

**i** *x*2 + 3*x* – 28 = 0 **j** *x*2 – 6*x* + 9 = 0

**k** 2*x*2 – 7*x* – 4 = 0 **l** 3*x*2 – 13*x* – 10 = 0

**2** Solve

**Hint**

Get all terms onto one side of the equation.

**a** *x*2 – 3*x* = 10 **b** *x*2 – 3 = 2*x*

**c** *x*2 + 5*x* = 24 **d** *x*2 – 42 = *x*

**e** *x*(*x* + 2) = 2*x* + 25 **f** *x*2 – 30 = 3*x* – 2

**g** *x*(3*x* + 1) = *x*2 + 15 **h** 3*x*(*x* – 1) = 2(*x* + 1)

**Solving quadratic equations by completing the square**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Completing the square lets you write a quadratic equation in the form *p*(*x* + *q*)2 + *r* = 0*.*

Examples

**Example 5** Solve *x*2 + 6*x* + 4 = 0. Give your solutions in surd form.

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| --- | --- |
| *x*2 + 6*x* + 4 = 0  (*x* + 3)2 − 9 + 4 = 0  (*x* + 3)2 − 5 = 0  (*x* + 3)2 = 5  *x* + 3 =  *x* =  So *x* =  or *x* = | **1** Write *x*2 + *bx* + *c* = 0 in the form  **2** Simplify.  **3** Rearrange the equation to work out *x*. First, add 5 to both sides.  **4** Square root both sides.  Remember that the square root of a value gives two answers.  **5** Subtract 3 from both sides to solve the equation.  **6** Write down both solutions. |

**Example 6** Solve 2*x*2 − 7*x* + 4 = 0. Give your solutions in surd form.

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| --- | --- |
| 2*x*2 − 7*x* + 4 = 0  = 0  = 0  = 0  = 0          So  or | **1** Before completing the square write *ax*2 + *bx* + *c* in the form  **2** Now complete the square by writing  in the form  **3** Expand the square brackets.  **4** Simplify.  *(continued on next page)*  **5** Rearrange the equation to work out *x*. First, add  to both sides.  **6** Divide both sides by 2.  **7** Square root both sides. Remember that the square root of a value gives two answers.  **8** Add  to both sides.  **9** Write down both the solutions. |

Practice

**3** Solve by completing the square.

**a** *x*2 – 4*x* – 3 = 0 **b** *x*2 – 10*x* + 4 = 0

**c** *x*2 + 8*x* – 5 = 0 **d** *x*2 – 2*x* – 6 = 0

**e** 2*x*2 + 8*x* – 5 = 0 **f** 5*x*2 + 3*x* – 4 = 0

**4** Solve by completing the square.

**Hint**

Get all terms onto one side of the equation.

**a** (*x* – 4)(*x* + 2) = 5

**b** 2*x*2 + 6*x* – 7 = 0

**c** *x*2 – 5*x* + 3 = 0

**Solving quadratic equations by using the formula**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Any quadratic equation of the form *ax*2 + *bx* + *c* = 0 can be solved using the formula 
* If *b*2 – 4*ac* is negative then the quadratic equation does not have any real solutions.
* It is useful to write down the formula before substituting the values for *a*, *b* and *c*.

Examples

**Example 7** Solve *x*2 + 6*x* + 4 = 0. Give your solutions in surd form.

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| --- | --- |
| *a* = 1, *b* = 6, *c* = 4            So  or | **1** Identify *a*, *b* and *c* and write down the formula.  Remember that  is all over 2*a*, not just part of it.  **2** Substitute *a* = 1, *b* = 6, *c* = 4 into the formula.  **3** Simplify. The denominator is 2, but this is only because *a* = 1. The denominator will not always be 2.  **4** Simplify .  **5** Simplify by dividing numerator and denominator by 2.  **6** Write down both the solutions. |

**Example 8** Solve 3*x*2 − 7*x* − 2 = 0. Give your solutions in surd form.

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| --- | --- |
| *a* = 3, *b* = −7, *c* = −2        So  or | **1** Identify *a*, *b* and *c*, making sure you get the signs right and write down the formula.  Remember that  is all over 2*a*, not just part of it.  **2** Substitute *a* = 3, *b* = −7, *c* = −2 into the formula.  **3** Simplify. The denominator is 6 when *a* = 3. A common mistake is to always write a denominator of 2.  **4** Write down both the solutions. |

Practice

**5** Solve, giving your solutions in surd form.

**a** 3*x*2 + 6*x* + 2 = 0 **b** 2*x*2 – 4*x* – 7 = 0

**6** Solve the equation *x*2 – 7*x* + 2 = 0

Give your solutions in the form , where *a*, *b* and *c* are integers.

**7** Solve 10*x*2 + 3*x* + 3 = 5

**Hint**

Get all terms onto one side of the equation.

Give your solution in surd form.

Extend

**8** Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

**a** 4*x*(*x* – 1) = 3*x* – 2

**b** 10 = (*x* + 1)2

**c** *x*(3*x* – 1) = 10

Answers

**1 a** *x* = 0 or *x* =  **b** *x* = 0 or *x* = 

**c** *x* = –5 or *x* = –2 **d** *x* = 2 or *x* = 3

**e** *x* = –1 or *x* = 4 **f** *x* = –5 or *x* = 2

**g** *x* = 4 or *x* = 6 **h** *x* = –6 or *x* = 6

**i** *x* = –7 or *x* = 4 **j** *x* = 3

**k** *x* =  or *x* = 4 **l** *x* =  or *x* = 5

**2 a** *x* = –2 or *x* = 5 **b** *x* = –1 or *x* = 3

**c** *x* = –8 or *x* = 3 **d** *x* = –6 or *x* = 7

**e** *x* = –5 or *x* = 5 **f** *x* = –4 or *x* = 7

**g** *x* = –3 or *x* = 2 **h** *x* =  or *x* = 2

**3 a** *x* = 2 + or *x* = 2 –  **b** *x* = 5 +  or *x* = 5 – 

**c** *x* = –4 +  or *x* = –4 –  **d** *x* = 1 +  or *x* = 1 – 

**e** *x* = –2 +  or *x* = –2 –  **f** *x* =  or *x* = 

**4 a** *x* = 1 +  or *x* = 1 –  **b** *x* =  or *x* = 

**c** *x* =  or *x* = 

**5 a** *x* = –1 +  or *x* = –1 –  **b** *x* = 1 +  or *x* = 1 – 

**6** *x* =  or *x* = 

**7** *x* =  or *x* = 

**8 a** *x* =  or *x* = 

**b** *x* = –1 +  or *x* = –1 – 

**c** *x* = –1 or *x* = 2