**Solving linear and quadratic simultaneous equations**

Key points

* Make one of the unknowns the subject of the linear equation (rearranging where necessary).
* Use the linear equation to substitute into the quadratic equation.
* There are usually two pairs of solutions.

Examples

**Example 1** Solve the simultaneous equations *y* = *x* + 1 and *x*2 + *y*2 = 13

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| *x*2 + (*x* + 1)2 = 13  *x*2 + *x*2 + *x* + *x* + 1 = 13  2*x*2 + 2*x* + 1 = 13  2*x*2 + 2*x* − 12 = 0  (2*x* − 4)(*x* + 3) = 0  So *x* = 2 or *x* = −3  Using *y* = *x* + 1  When *x* = 2, *y* = 2 + 1 = 3  When *x* = −3, *y* = −3 + 1 = −2  So the solutions are  *x* = 2, *y* = 3 and *x* = −3, *y* = −2  Check:  equation 1: 3 = 2 + 1 YES  and −2 = −3 + 1 YES  equation 2: 22 + 32 = 13 YES  and (−3)2 + (−2)2 = 13 YES | **1** Substitute *x* + 1 for *y* into the second equation.  **2** Expand the brackets and simplify.  **3** Factorise the quadratic equation.  **4** Work out the values of *x*.  **5** To find the value of *y*, substitute both values of *x* into one of the original equations.  **6** Substitute both pairs of values of *x* and *y* into both equations to check your answers. |

**Example 2** Solve 2*x* + 3*y* = 5 and 2*y*2 + *xy* = 12 simultaneously.

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| (*y* + 8)(*y* − 3) = 0  So *y* = −8 or *y* = 3  Using 2*x* + 3*y* = 5  When *y* = −8, 2*x* + 3 × (−8) = 5, *x* = 14.5  When *y* = 3, 2*x* + 3 × 3 = 5, *x* = −2  So the solutions are  *x* = 14.5, *y* = −8 and *x* = −2, *y* = 3  Check:  equation 1: 2 × 14.5 + 3 × (−8) = 5 YES  and 2 × (−2) + 3 × 3 = 5 YES  equation 2: 2×(−8)2 + 14.5×(−8) = 12 YES  and 2 × (3)2 + (−2) × 3 = 12 YES | **1** Rearrange the first equation.  **2** Substitute  for *x* into the second equation. Notice how it is easier to substitute for *x* than for *y*.  **3** Expand the brackets and simplify.  **4** Factorise the quadratic equation.  **5** Work out the values of *y*.  **6** To find the value of *x*, substitute both values of *y* into one of the original equations.  **7** Substitute both pairs of values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**1** *y* = 2*x* + 1 **2** *y* = 6 − *x*

*x*2 + *y*2 = 10 *x*2 + *y*2 = 20

**3** *y* = *x* – 3 **4** *y* = 9 − 2*x*

*x*2 + *y*2 = 5 *x*2 + *y*2 = 17

**5** *y* = 3*x* – 5 **6** *y* = *x* − 5

*y* = *x*2 − 2*x* + 1 *y* = *x*2 − 5*x* − 12

**7** *y* = *x* + 5 **8** *y* = 2*x* – 1

*x*2 + *y*2 = 25 *x*2 + *xy* = 24

**9** *y* = 2*x* **10** 2*x* + *y* = 11

*y*2 – *xy* = 8 *xy* = 15

Extend

**11** *x* – *y* = 1 **12** *y* – *x* = 2

*x*2 + *y*2 = 3 *x*2 + *xy* = 3

Answers

**1** *x* = 1, *y* = 3



**2** *x* = 2, *y* = 4

*x* = 4, *y* = 2

**3** *x* = 1, *y* = −2

*x* = 2, *y* = –1

**4** *x* = 4, *y* = 1



**5** *x* = 3, *y* = 4

*x* = 2, *y* = 1

**6** *x* = 7, *y* = 2

*x* = −1, *y* = −6

**7** *x* = 0, *y* = 5

*x* = –5, *y* = 0

**8** *x* = , *y* = 

*x* = 3, *y* = 5

**9** *x* = –2, *y* = –4

*x* = 2, *y* = 4

**10** *x* = , *y* = 6

*x* = 3, *y* = 5

**11** *x* = , *y* = 

*x* = , *y* = 

**12** *x* = , *y* = 

*x* = , *y* = 