**Solving linear simultaneous equations using the elimination method**

Key points

* Two equations are simultaneous when they are both true at the same time.
* Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
* Make sure that the coefficient of one of the unknowns is the same in both equations.
* Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

**Example 1** Solve the simultaneous equations 3*x* + *y* = 5 and *x* + *y* = 1

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| --- | --- |
|  3*x* + *y* = 5*– x* + *y* = 1  2*x* = 4So *x* = 2Using *x* + *y* = 1 2 + *y* = 1So *y* = −1Check: equation 1: 3 × 2 + (−1) = 5 YES equation 2: 2 + (−1) = 1 YES | **1** Subtract the second equation from the first equation to eliminate the *y* term.**2** To find the value of *y*, substitute *x*= 2 into one of the original equations.**3** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 2** Solve *x* + 2*y* = 13 and 5*x* − 2*y* = 5 simultaneously.

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| --- | --- |
|  *x* + 2*y* = 13+ 5*x* − 2*y* = 5     6*x* = 18So *x* = 3Using *x* + 2*y* = 13 3 + 2*y* = 13So *y* = 5Check: equation 1: 3 + 2 × 5 = 13 YES equation 2: 5 × 3 − 2 × 5 = 5 YES | **1** Add the two equations together to eliminate the *y* term.**2** To find the value of *y*, substitute *x*= 3 into one of the original equations.**3** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 3** Solve 2*x* + 3*y* = 2 and 5*x* + 4*y* = 12 simultaneously.

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| --- | --- |
| (2*x* + 3*y* = 2) × 4  8*x* + 12*y* = 8(5*x* + 4*y* = 12) × 3 15*x* + 12*y* = 36  7*x* = 28So *x* = 4Using 2*x* + 3*y*  = 2 2 × 4 + 3*y* = 2So *y* = −2Check: equation 1: 2 × 4 + 3 × (−2) = 2 YES equation 2: 5 × 4 + 4 × (−2) = 12 YES | **1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of *y* the same for both equations. Then subtract the first equation from the second equation to eliminate the *y* term.**2** To find the value of *y*, substitute *x*= 4 into one of the original equations.**3** Substitute the values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**1** 4*x* + *y* = 8 **2** 3*x* + *y* = 7

 *x* + *y* = 5 3*x* + 2*y* = 5

**3** 4*x* + *y* = 3 **4** 3*x* + 4*y* = 7

 3*x* – *y* = 11 *x* – 4*y* = 5

**5** 2*x* + *y* = 11 **6** 2*x* + 3*y* = 11

 *x* – 3*y* = 9 3*x* + 2*y* = 4

**Solving linear simultaneous equations using the substitution method**

 **A LEVEL LINKS**

 **Scheme of work:** 1c. Equations – quadratic/linear simultaneous

 **Textbook:**Pure Year 1, 3.1 Linear simultaneous equations

Key points

* The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

**Example 4** Solve the simultaneous equations *y* = 2*x* + 1 and 5*x* + 3*y* = 14

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| --- | --- |
| 5*x* + 3(2*x* + 1) = 145*x* + 6*x* + 3 = 1411*x* + 3 = 1411*x* = 11So *x* = 1Using *y* = 2*x* + 1 *y* = 2 × 1 + 1So *y* = 3Check: equation 1: 3 = 2 × 1 + 1 YES equation 2: 5 × 1 + 3 × 3 = 14 YES | **1** Substitute 2*x* + 1 for *y* into the second equation.**2** Expand the brackets and simplify.**3** Work out the value of *x*.**4** To find the value of *y*, substitute *x*= 1 into one of the original equations.**5** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 5** Solve 2*x* − *y* = 16 and 4*x* + 3*y* = −3 simultaneously.

|  |  |
| --- | --- |
| *y* = 2*x* − 164*x* + 3(2*x* − 16) = −34*x* + 6*x* − 48 = −310*x* − 48 = −310*x* = 45So *x* =  Using *y* = 2*x* − 16 *y* = 2 ×  − 16So *y* = −7Check: equation 1: 2 ×  – (–7) = 16 YES equation 2: 4 ×  + 3 × (−7) = −3 YES | **1** Rearrange the first equation.**2** Substitute 2*x* − 16 for *y* into the second equation.**3** Expand the brackets and simplify.**4** Work out the value of *x*.**5** To find the value of *y*, substitute *x*=  into one of the original equations.**6** Substitute the values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**7** *y* = *x* –4 **8** *y* = 2*x* – 3

 2*x* + 5*y* = 43 5*x* – 3*y* = 11

**9** 2*y* = 4*x* + 5 **10** 2*x* = *y* – 2

 9*x* + 5*y* = 22 8*x* – 5*y* = –11

**11** 3*x* + 4*y* = 8 **12** 3*y* = 4*x* – 7

 2*x* – *y* = –13 2*y* = 3*x* – 4

**13** 3*x* = *y* – 1 **14** 3*x* + 2*y* + 1 = 0

 2*y* – 2*x* = 3 4*y* = 8 – *x*

Extend

**15** Solve the simultaneous equations 3*x* + 5*y* − 20 = 0 and .

Answers

**1** *x* = 1, *y* = 4

**2** *x* = 3, *y* = –2

**3** *x* = 2, *y* = –5

**4** *x* = 3, *y* = –

**5** *x* = 6, *y* = –1

**6** *x* = –2, *y* = 5

**7** *x* = 9, *y* = 5

**8** *x* = –2, *y* = –7

**9** *x* = , *y* = 3

**10** *x* = , *y* = 3

**11** *x* = –4, *y* = 5

**12** *x* = –2, *y* = –5

**13** *x* = , *y* = 1

**14** *x* = –2, *y* = 2

**15** *x* = –2, *y* = 5