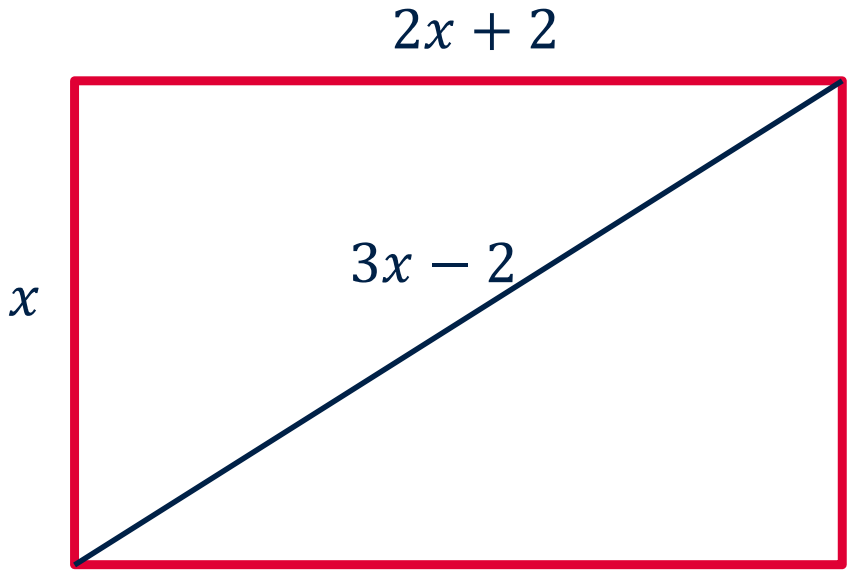
Quadthagoras

Find the length, width and diagonal of this rectangle

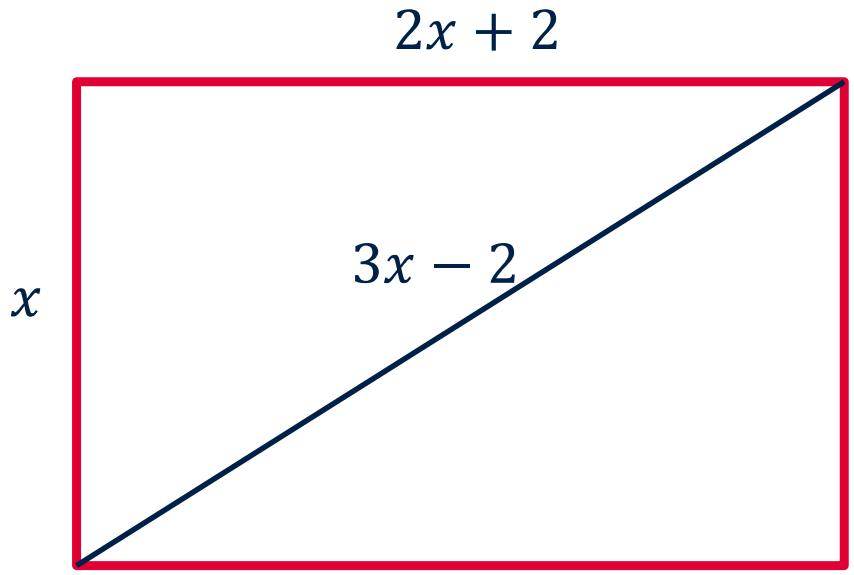


Quadthagoras

Solutions on the next slide….

Quadthagoras Solution

Find the length, width and diagonal of this rectangle



By Pythagoras' Theorem:

$$x^2 + (2x + 2)^2 = (3x - 2)^2$$
$$x^2 + 4x^2 + 8x + 4 = 9x^2 - 12x + 4$$
$$4x^2 - 20x = 0$$
$$4x(x - 5) = 0$$
$$x = 0 \text{ or } x = 5$$

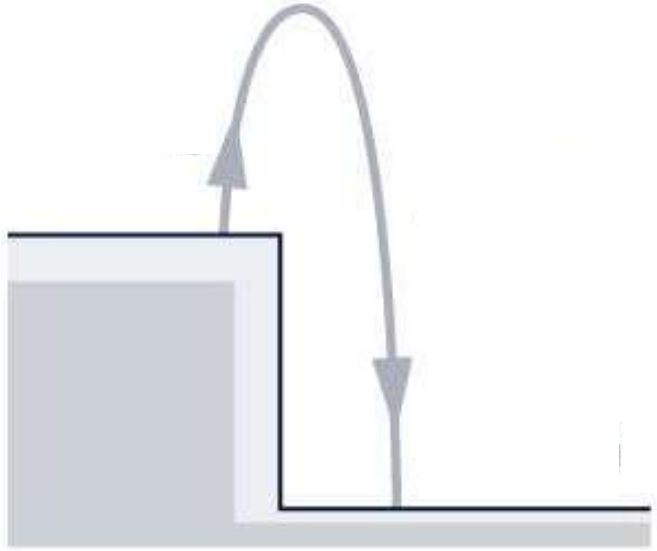
- As we are finding lengths, only $x = 5$ makes sense in this context.
- Therefore suitable lengths are 5, 12 and 13

Up in the air!

An object is launched from a cliff that is $58.8m$ high.
The speed of the object is 19.6 metres per second (m/s).

The equation for the object's height h above the ground at time t seconds after launch is $h = -4.9t^2 + 19.6t + 58.8$ where h is in metres.

- When does the object strike the ground?



Up in the air!

Solutions on the next slide….

Up in the air Solution

An object is launched from a cliff that is $58.8m$ high.
The speed of the object is 19.6 metres per second (m/s).

The equation for the object's height h above the ground at time t seconds after launch is

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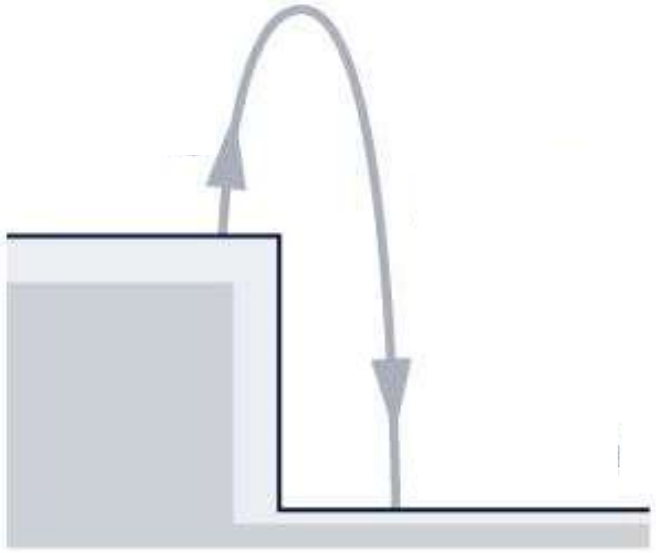

The object will hit the ground when $h = 0$
So we need to solve $0 = -4.9t^2 + 19.6t + 58.8$

Tip: rearrange to make t^2 positive

$$4.9t^2 - 19.6t - 58.8 = 0$$

Tip: 4.9 is a factor of 19.6 and 58.8

There are other methods you can use to solve this equation

$$t^2 - 4t - 12 = 0$$

$$(t - 6)(t + 2) = 0$$

$t = 6$ *or* $t = -2$ the object strikes the ground after 6 seconds

The answer is a positive as it represents the time after launch

Which Way?

In the skills check you saw how we can solve quadratic equations by factorising or completing the square.

We can also use the quadratic formula, for a quadratic $ax^2 + bx + c = 0$ the solutions are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Try solving $x^2 + 4x - 21 = 0$ using each of the three methods.

Try solving $3x^2 + 4x - 2 = 0$ using each of the three methods.

Which Way?

Solutions on the next slide….

Which Way? Solutions

Solve by

$$x^2 + 4x - 21 = 0$$

Factorising

$$(x + 7)(x - 3) = 0$$

$x = -7$ or $x = 3$

Which of the methods would be your first choice here?

Completing the square

$$(x + 2)^2 - 4 - 21 = 0$$
$$(x + 2)^2 = 25$$
$$(x + 2) = \pm 5$$

$x = -7$ or $x = 3$

Was it different when you tried to solve $3x^2 + 4x - 2 = 0$?

Quadratic Formula

$$a = 1, \mathrm{b} = 4, \mathrm{c} = -21$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-21)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16 + 84}}{2}$$
$$= \frac{-4 \pm \sqrt{100}}{2}$$
$$= \frac{-4 \pm 10}{2}$$
$$= -2 \pm 5$$

$x = -7$ or $x = 3$

Which Way? Solutions

Solve by

$$3x^2 + 4x - 2 = 0$$

Factorising

It doesn't factorise

Horrible!

Completing the square

$$3[x^2 + \frac{4}{3}x - \frac{2}{3}] = 0$$

$$3[\left(x + \frac{2}{3}\right)^2 - \frac{4}{9} - \frac{2}{3}] = 0$$

$$3[\left(x + \frac{2}{3}\right)^2 - \frac{10}{9}] = 0$$

$$3\left(x + \frac{2}{3}\right)^2 - \frac{30}{9} = 0$$

$$3\left(x + \frac{2}{3}\right)^2 = \frac{30}{9}$$

$$\left(x + \frac{2}{3}\right)^2 = \frac{10}{9}$$

$$\left(x + \frac{2}{3}\right) = \pm\sqrt{\frac{10}{9}}$$

$x = -\frac{2}{3} + \frac{\sqrt{10}}{3}$ or $x = -\frac{2}{3} - \frac{\sqrt{10}}{3}$

Quadratic Formula

$$a = 3, \mathrm{b} = 4, \mathrm{c} = -2$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{16 + 24}}{6}$$

$$= \frac{-4 \pm \sqrt{40}}{6}$$

$$= -\frac{2}{3} \pm \frac{\sqrt{10}}{3}$$

or $x = -\frac{2}{3} + \frac{\sqrt{10}}{3}$

$x = -\frac{2}{3} - \frac{\sqrt{10}}{3}$