

LPGS GCSE March Mock Paper 1

HIGHER TIER

Tuesday 12th March 2019 AFTERNOONName KGMaths Teacher Worked Solutions**Instructions**

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may not be used.**

**Information**

- The total mark for this paper is 80.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

TOTAL MARK out 80 _____

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Work out the value of $\frac{3^7 \times 3^{-2}}{3^3} = \frac{3^5}{3^3}$
 $= 3^2$
 $= 9$

9

(Total for Question 1 is 2 marks)

2 $v^2 = u^2 + 2as$

$u = 12$ $a = -3$ $s = 18$

(a) Work out a value of v . $v^2 = 12^2 + 2 \times -3 \times 18$
 $= 144 - 108$
 $= 36$
 $v = 6 \text{ or } v = -6$

6 or -6
(2)

(b) Make s the subject of $v^2 = u^2 + 2as$

$$\begin{aligned} v^2 &= u^2 + 2as \\ -u^2] \quad v^2 - u^2 &= 2as \\ \div 2a] \quad \frac{v^2 - u^2}{2a} &= s \end{aligned}$$

$$s = \frac{v^2 - u^2}{2a}$$

(2)

(Total for Question 2 is 4 marks)



- 3 A bonus of £2100 is shared by 10 people who work for a company.
40% of the bonus is shared equally between 3 managers.
The rest of the bonus is shared equally between 7 salesmen.

One of the salesmen says,

"If the bonus is shared equally between all 10 people I will get 25% more money."

Is the salesman correct?

You must show how you get your answer.

managers : $10\% \text{ of } £2100 = £210$
 $40\% \text{ of } £2100 = £210 \times 4 = £840$

Salesmen : $£2100 - £840 = £1260$

$$\begin{array}{r} 180 \\ 7 \overline{)1260} \end{array} \quad \text{each salesman gets } £180.$$

shared equally : $£2100 \div 10 = £210$
each salesman gets £210.

shared equally, the salesman gets £30 more

$$25\% \text{ of } £180 = £45$$

The salesman is incorrect.

(Total for Question 3 is 5 marks)

4 It would take 120 minutes to fill a swimming pool using water from 5 taps.

(a) How many minutes will it take to fill the pool if only 3 of the taps are used?

$$120 \times 5 = 600$$

$$600 \div 3 = 200$$

200 minutes

(2)

(b) State one assumption you made in working out your answer to part (a).

All taps run at the same rate.

(1)

(Total for Question 4 is 3 marks)

5 A plane travels at a speed of 213 miles per hour.

(a) Work out an estimate for the number of seconds the plane takes to travel 1 mile.

$$\begin{aligned} \text{seconds in one hour} &= 60 \times 60 \\ &= 3600 \end{aligned}$$

$$\frac{3600}{200} = 18$$

18 seconds

(3)

(b) Is your answer to part (a) an underestimate or an overestimate?
Give a reason for your answer.

An overestimate. I have underestimated the speed.

(1)

(Total for Question 5 is 4 marks)



6 Solve the simultaneous equations

$$5x + y = 21 \quad \textcircled{1}$$

$$x - 3y = 9 \quad \textcircled{2}$$

$$\begin{array}{r} \textcircled{1} \times \textcircled{3}: \quad 15x + 3y = 63 \\ \quad \quad \quad x - 3y = 9 \\ \hline 16x \quad \quad = 72 \end{array}$$

$$x = \frac{72}{16} = \frac{9}{2} = 4.5$$

substitute into $\textcircled{2}$: $4.5 - 3y = 9$

$$-4.5] \quad -3y = 4.5$$

$$\div -3] \quad y = -1.5$$

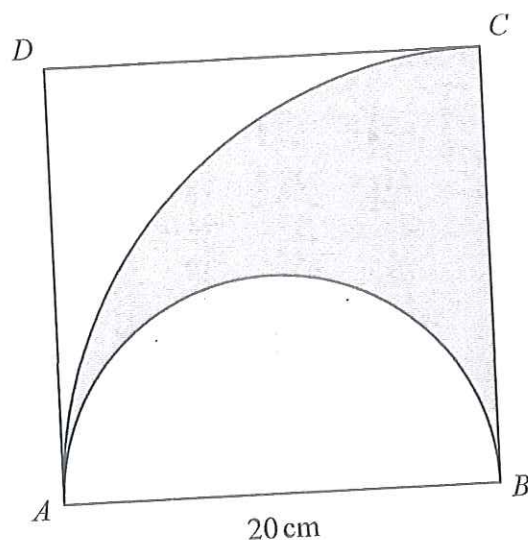
$$x = 4.5$$

$$y = -1.5$$

(Total for Question 6 is 3 marks)



- 7 The diagram shows a square $ABCD$ with sides of length 20 cm. It also shows a semicircle and an arc of a circle.



AB is the diameter of the semicircle.
 AC is an arc of a circle with centre B .

Show that $\frac{\text{area of shaded region}}{\text{area of square}} = \frac{\pi}{8}$

$$\begin{aligned}\text{Area of sector } ABC &= \frac{1}{4} \times \pi \times 20^2 \\ &= \frac{1}{4} \times \pi \times 400 \\ &= 100\pi\end{aligned}$$

$$\begin{aligned}\text{Area of semicircle} &= \frac{1}{2} \times \pi \times 10^2 \\ &= \frac{1}{2} \times \pi \times 100 \\ &= 50\pi\end{aligned}$$

$$\begin{aligned}\text{Shaded area} &= 100\pi - 50\pi \\ &= 50\pi\end{aligned}\quad \begin{aligned}\text{Area of square} &= 20 \times 20 \\ &= 400\end{aligned}$$

$$\frac{\text{area of shaded region}}{\text{area of square}} = \frac{50\pi}{400} = \frac{\pi}{8}$$

(Total for Question 7 is 4 marks)

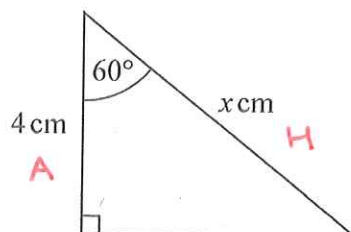


- 8 (a) Write down the exact value of $\tan 45^\circ$

1

(1)

Here is a right-angled triangle.



$$\cos 60^\circ = 0.5$$

- (b) Work out the value of x .



$$H = \frac{A}{C}$$

$$x = \frac{4}{\cos 60}$$

$$= \frac{4}{0.5}$$

$$= 8$$

8

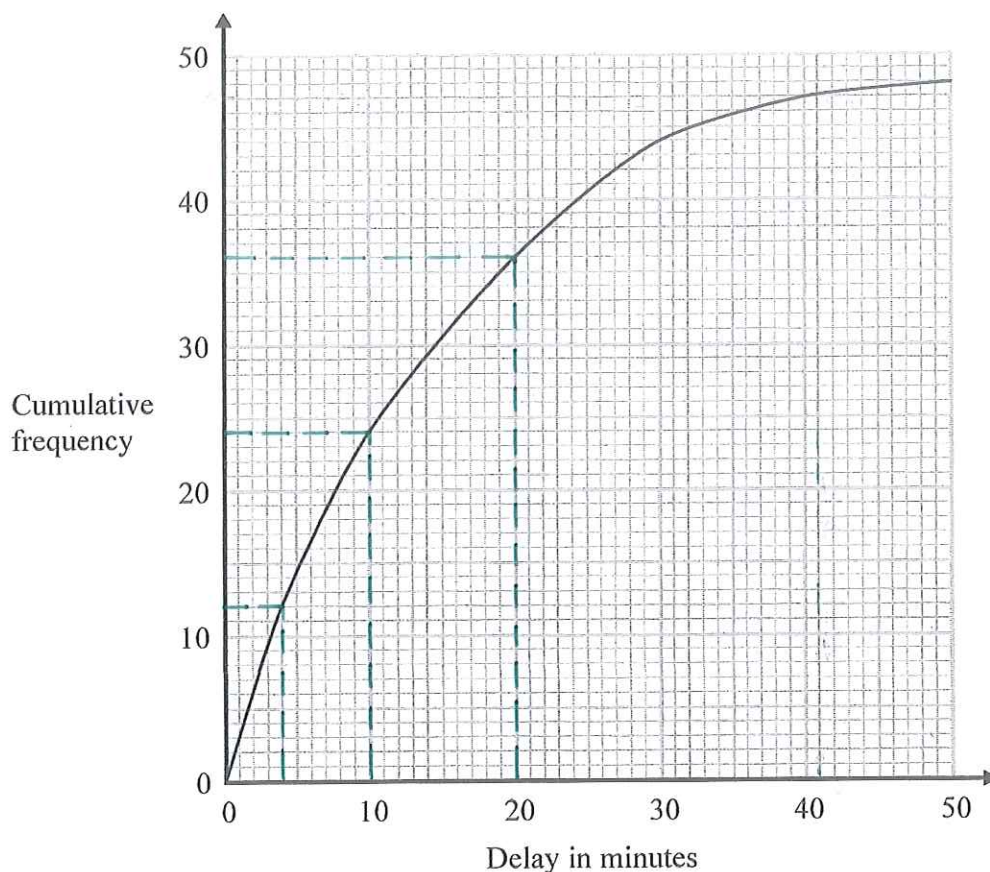
(2)

(Total for Question 8 is 3 marks)



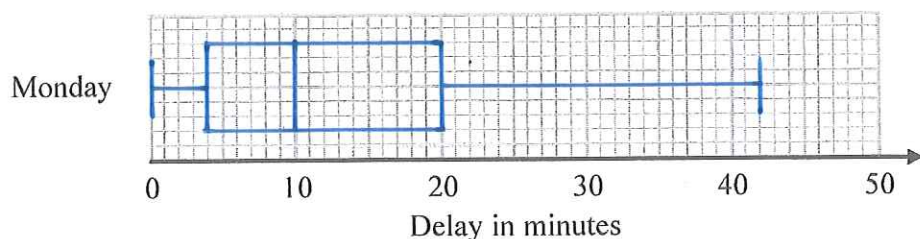
- 9 The times that 48 trains left a station on Monday were recorded.

The cumulative frequency graph gives information about the numbers of minutes the trains were delayed, correct to the nearest minute.



The shortest delay was 0 minutes.
The longest delay was 42 minutes.

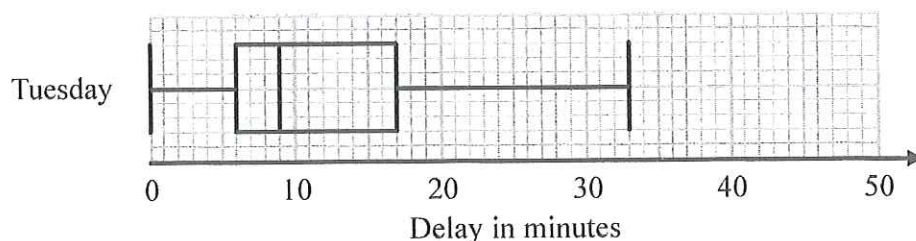
- (a) On the grid below, draw a box plot for the information about the delays on Monday.



(3)

48 trains left the station on Tuesday.

The box plot below gives information about the delays on Tuesday.



- (b) Compare the distribution of the delays on Monday with the distribution of the delays on Tuesday.

In general the trains were more delayed on Monday, with a median delay of 10 minutes compared to 9 minutes on Tuesday. The delays were also more spread on Monday with a range of 42 minutes compared to 33 minutes on Tuesday.

(2)

Mary says,

"The longest delay on Tuesday was 33 minutes.

This means that there must be some delays of between 25 minutes and 30 minutes."

- (c) Is Mary right?

You must give a reason for your answer.

She is wrong. There must be some delays of between 17 and 33 minutes but not necessarily between 25 and 30 minutes.

(1)

(Total for Question 9 is 6 marks)

10 (a) Simplify $\frac{x-1}{5(x-1)^2}$

$$\frac{1}{5(x-1)}$$

(1)

(b) Factorise fully $50 - 2y^2$

$$\begin{aligned} 50 - 2y^2 &= 2(25 - y^2) \\ &= 2(5+y)(5-y) \end{aligned}$$

$$2(5+y)(5-y)$$

(2)

(Total for Question 10 is 3 marks)

11 Jack and Sadia work for a company that sells boxes of breakfast cereal.

The company wants to have a special offer.

Here is Jack's idea for the special offer.

Put 25% more cereal into each box and do **not** change the price.

Here is Sadia's idea.

Reduce the price and do **not** change the amount of cereal in each box.

Sadia wants her idea to give the same value for money as Jack's idea.

By what percentage does she need to reduce the price?

e.g. original box: 100g for £1.

Jack: 125g for £1

$$\frac{125}{100} = 1.25 \text{ g per p}$$

Sadia 1.25g per p

$$\frac{100}{1.25} = \frac{10000}{125} = 80\text{p}$$

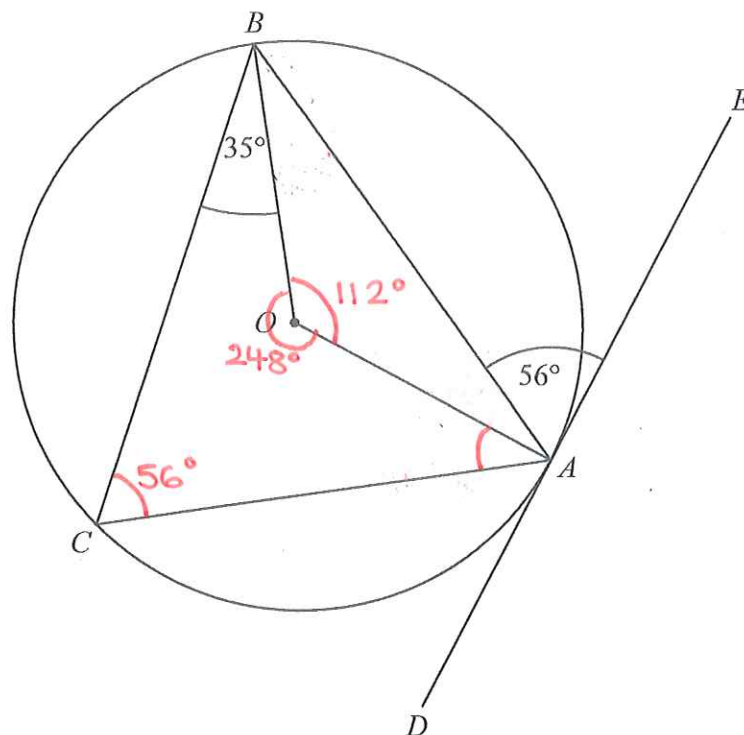
she would sell 100g for 80p, a reduction of 20%.

20 %

(Total for Question 11 is 3 marks)



12



A, B and C are points on the circumference of a circle, centre O.
DAE is the tangent to the circle at A.

Angle BAE = 56°

Angle CBO = 35°

Work out the size of angle CAO.
You must show all your working.

angle ACB = 56° (alternate segment theorem.)

angle AOB = 112° (angle at centre is double angle at circumference.)

reflex angle AOB = 360 - 112 = 248° (angles in a full turn add to 360°)

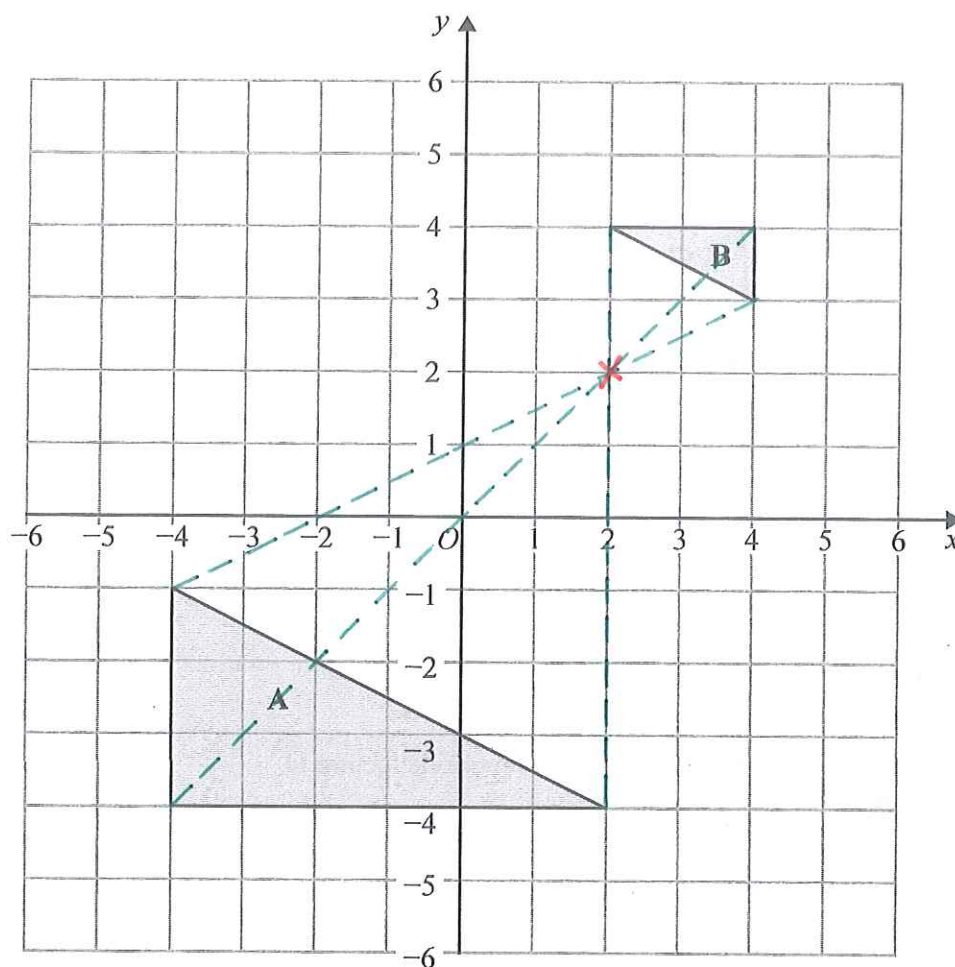
angle CAO = 360 - 248 - 35 - 56 = 21° (angles in quadrilateral AOCB add to 360°)

21

(Total for Question 12 is 3 marks)



13



Describe fully the single transformation that maps triangle A onto triangle B.

Enlargement, scale factor $-\frac{1}{3}$, centre $(2, 2)$.

(Total for Question 13 is 2 marks)

14 (a) Work out the value of $\left(\frac{16}{81}\right)^{\frac{3}{4}} = \frac{16^{3/4}}{81^{3/4}}$

$$= \frac{(4\sqrt{16})^3}{(4\sqrt{81})^3}$$

$$= \frac{2^3}{3^3}$$

$$= \frac{8}{27}$$

$$\frac{8}{27}$$

(2)

$$3^a = \frac{1}{9} \quad 3^b = 9\sqrt{3} \quad 3^c = \frac{1}{\sqrt{3}}$$

(b) Work out the value of $a + b + c$

$$\frac{1}{9} = \frac{1}{3^2}$$

$$= 3^{-2}$$

$$a = -2$$

$$9\sqrt{3} = (\sqrt{3})^5$$

$$= 3^{\frac{5}{2}}$$

$$b = \frac{5}{2}$$

$$\frac{1}{\sqrt{3}} = \frac{1}{3^{1/2}}$$

$$= 3^{-1/2}$$

$$c = -\frac{1}{2}$$

$$a + b + c = -2 + \frac{5}{2} - \frac{1}{2}$$

$$= 0$$

$$0$$

(2)

(Total for Question 14 is 4 marks)



15 Three solid shapes A, B and C are similar.

The surface area of shape A is 4 cm^2

The surface area of shape B is 25 cm^2

The ratio of the volume of shape B to the volume of shape C is 27:64

Work out the ratio of the height of shape A to the height of shape C.

Give your answer in its simplest form.

$$\text{Area scale factor } A \rightarrow B = \frac{25}{4}$$

$$\text{length scale factor } A \rightarrow B = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

$$\text{Volume scale factor } B \rightarrow C = \frac{64}{27}$$

$$\text{length scale factor } B \rightarrow C = \sqrt[3]{\frac{64}{27}} = \frac{4}{3}$$

$$\text{length scale factor } A \rightarrow C = \frac{5}{2} \times \frac{4}{3} = \frac{20}{6} = \frac{10}{3}$$

\therefore ratio of heights is 3:10

3:10

(Total for Question 15 is 4 marks)

16 Prove algebraically that $0.2\dot{5}\dot{6}$ can be written as $\frac{127}{495}$

$$\text{Let } r = 0.256565656 \dots$$

$$100r = 25.656565656 \dots$$

$$100r - r = 25.656565656 \dots - 0.256565656 \dots$$

$$99r = 25.4$$

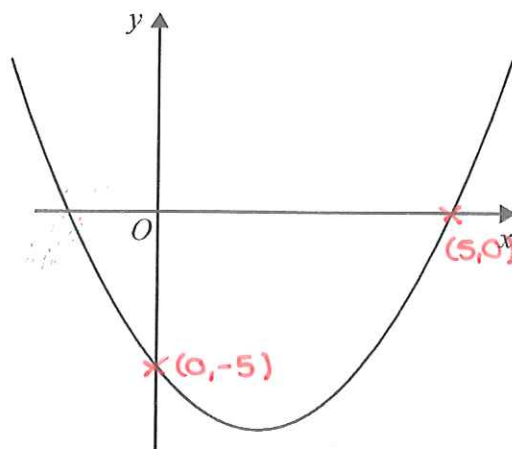
$$r = \frac{25.4}{99}$$

$$= \frac{254}{990}$$

$$= \frac{127}{495}$$

(Total for Question 16 is 3 marks)

17 Here is a sketch of a curve.



The equation of the curve is $y = x^2 + ax + b$ where a and b are integers.

The points $(0, -5)$ and $(5, 0)$ lie on the curve.

Find the coordinates of the turning point of the curve.

$$(0, -5): x=0, y=-5$$

$$-5 = 0^2 + a \times 0 + b$$

$$b = -5$$

$$(5, 0): x=5, y=0$$

$$0 = 5^2 + 5a - 5$$

$$= 20 + 5a$$

$$a = -4$$

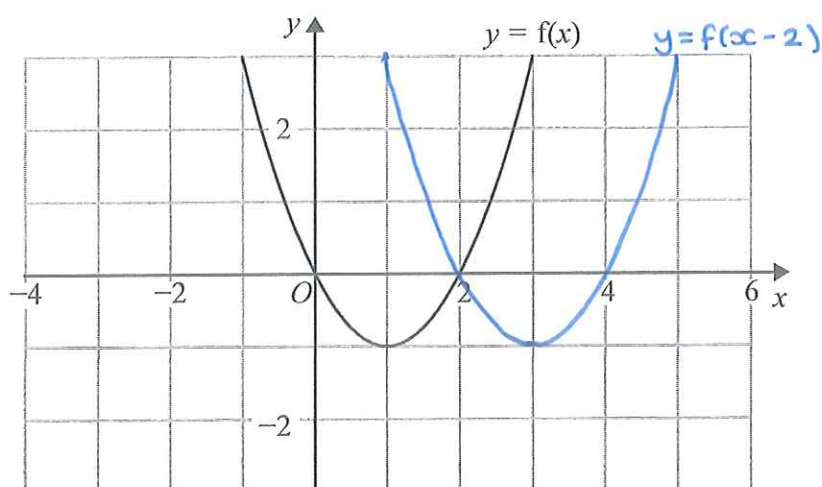
$$y = x^2 - 4a - 5$$

$$= (x-2)^2 - 9$$

(2, -9)

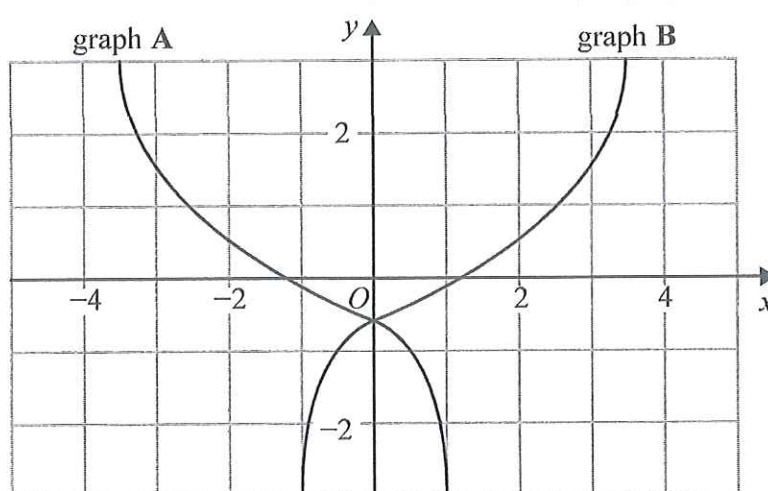
(Total for Question 17 is 4 marks)

18 The graph of $y = f(x)$ is shown on the grid below.



(a) On the grid above, sketch the graph of $y = f(x - 2)$

(1)



On the grid, graph A has been reflected to give graph B.

The equation of graph A is $y = g(x)$

(b) Write down the equation of graph B.

$$y = g(-x)$$

(1)

(Total for Question 18 is 2 marks)



19 For all values of x

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

(a) Show that $gf(x) = 2x(x + 2)$

$$\begin{aligned} gf(x) &= 2((x+1)^2 - 1) \\ &= 2(x^2 + 2x + 1 - 1) \\ &= 2(x^2 + 2x) \\ &= 2x(x + 2) \end{aligned}$$

(2)

(b) Find $g^{-1}(7)$

$$g(x) = 7$$

$$2(x - 1) = 7$$

$$\div 2] \quad x - 1 = 3.5$$

$$+1] \quad x = 4.5$$

4.5

(2)

(Total for Question 19 is 4 marks)

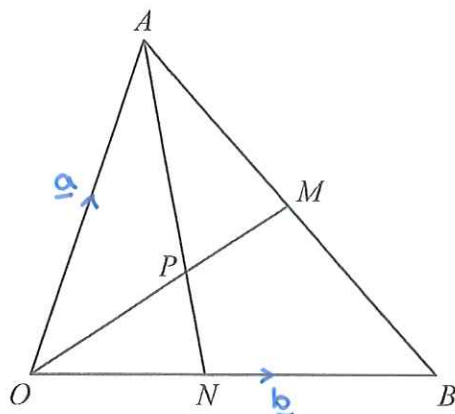
20 Show that $\frac{(\sqrt{18} + \sqrt{2})^2}{\sqrt{8} - 2}$ can be written in the form $a(b + \sqrt{2})$ where a and b are integers.

$$\begin{aligned}\frac{(\sqrt{18} + \sqrt{2})(\sqrt{18} + \sqrt{2})}{\sqrt{8} - 2} &= \frac{18 + \sqrt{36} + \sqrt{36} + 2}{\sqrt{8} - 2} \\&= \frac{18 + 6 + 6 + 2}{\sqrt{8} - 2} \\&= \frac{32}{\sqrt{8} - 2} \\&= \frac{32(\sqrt{8} + 2)}{(\sqrt{8} - 2)(\sqrt{8} + 2)} \\&= \frac{32\sqrt{8} + 64}{8 - 4} \\&= \frac{32\sqrt{8} + 64}{4} \\&= 8 \times 2\sqrt{2} + 16 \\&= 16\sqrt{2} + 16 \\&= 16(1 + \sqrt{2})\end{aligned}$$

(Total for Question 20 is 3 marks)



21



OAB is a triangle.

OPM and APN are straight lines.

M is the midpoint of AB .

$$\vec{OA} = \underline{a} \quad \vec{OB} = \underline{b}$$

$$OP:PM = 3:2$$

Work out the ratio $ON:NB$

$$\vec{AB} = \underline{b} - \underline{a}$$

$$\vec{AM} = \frac{1}{2}(\underline{b} - \underline{a})$$

$$\therefore \vec{OM} = \underline{a} + \frac{1}{2}(\underline{b} - \underline{a}) = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$$

$$\vec{OP} = \frac{3}{5}\vec{OM} = \frac{3}{10}\underline{a} + \frac{3}{10}\underline{b}$$

$$\vec{AP} = \vec{OP} - \underline{a} = -\frac{7}{10}\underline{a} + \frac{3}{10}\underline{b}$$

$$\vec{AN} = -\underline{a} + x\underline{b} \quad \text{and} \quad \vec{AN} = y\vec{AP} = -\frac{7}{10}y\underline{a} + \frac{3}{10}y\underline{b}$$

$$-\underline{a} + x\underline{b} = -\frac{7}{10}y\underline{a} + \frac{3}{10}y\underline{b}$$

$$-\frac{7}{10}y = -1 \quad y = \frac{10}{7}$$

$$x = \frac{3}{10} \times \frac{10}{7} = \frac{3}{7}$$

$$\therefore ON:NB \text{ is } 3:4$$

3:4

(Total for Question 21 is 5 marks)

22 There are only green pens and blue pens in a box.

There are three more blue pens than green pens in the box.
There are more than 12 pens in the box.

Simon is going to take at random two pens from the box.

The probability that Simon will take two pens of the same colour is $\frac{27}{55}$

Work out the number of green pens in the box.

Let b = number of blue pens
 g = number of green pens

$$\textcircled{1} \quad b = g + 3 \quad b + g > 12 \quad \textcircled{2}$$

$$P(b, b) + P(g, g) = \frac{27}{55}$$

$$\frac{b}{b+g} \times \frac{b-1}{b+g-1} + \frac{g}{b+g} \times \frac{g-1}{b+g-1} = \frac{27}{55}$$

$$\frac{b(b-1) + g(g-1)}{(b+g)(b+g-1)} = \frac{27}{55} \quad \textcircled{3}$$

substitute $\textcircled{1}$ into $\textcircled{3}$:
$$\frac{(g+3)(g+2) + g(g-1)}{(g+3+g)(g+3+g-1)} = \frac{27}{55}$$

$$\frac{g^2 + 5g + 6 + g^2 - g}{(2g+3)(2g+2)} = \frac{27}{55}$$

$$55(2g^2 + 4g + 6) = 27(2g+3)(2g+2)$$

$$110g^2 + 220g + 330 = 108g^2 + 270g + 162$$

$$2g^2 - 50g + 168 = 0$$

$$g^2 - 25g + 84 = 0$$

$$(g-4)(g-21) = 0$$

$$g = 4 \text{ or } g = 21$$

21

$$b + g > 12 \Rightarrow g = 21$$

(Total for Question 22 is 6 marks)

TOTAL FOR PAPER IS 80 MARKS

