WORKED SOLUTIONS

1 hour 30 mins

LPGS GCSE March Mock Paper 2

HIGHER TIER

Wednesday March 2019 MORNING

Name KG

Maths Teacher WORKED SOLUTIONS

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may be used.**
- If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

TOTAL MARK out 80 _____

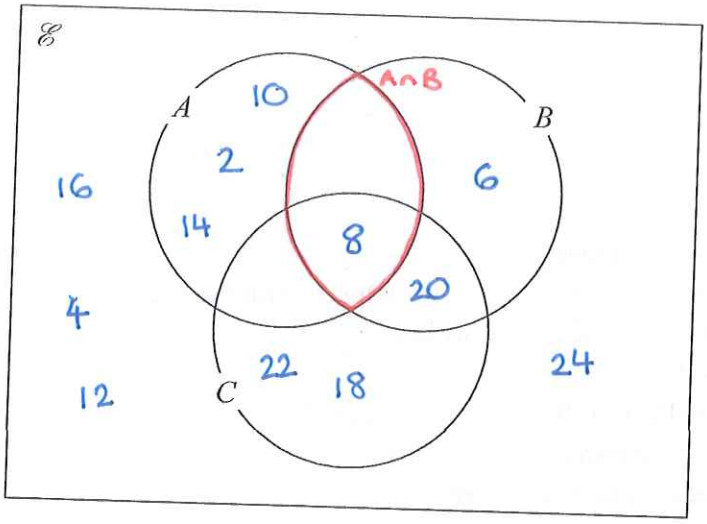
Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 $\mathscr{E}$ = {even numbers between 1 and 25}
A = {2, 8, 10, 14}
B = {6, 8, 20}
C = {8, 18, 20, 22}

(a) Complete the Venn diagram for this information.



(4)

A number is chosen at random from $\mathscr{E}$.

(b) Find the probability that the number is a member of $A \cap B$.

$\frac{1}{12}$

(2)

(Total for Question 1 is 6 marks)

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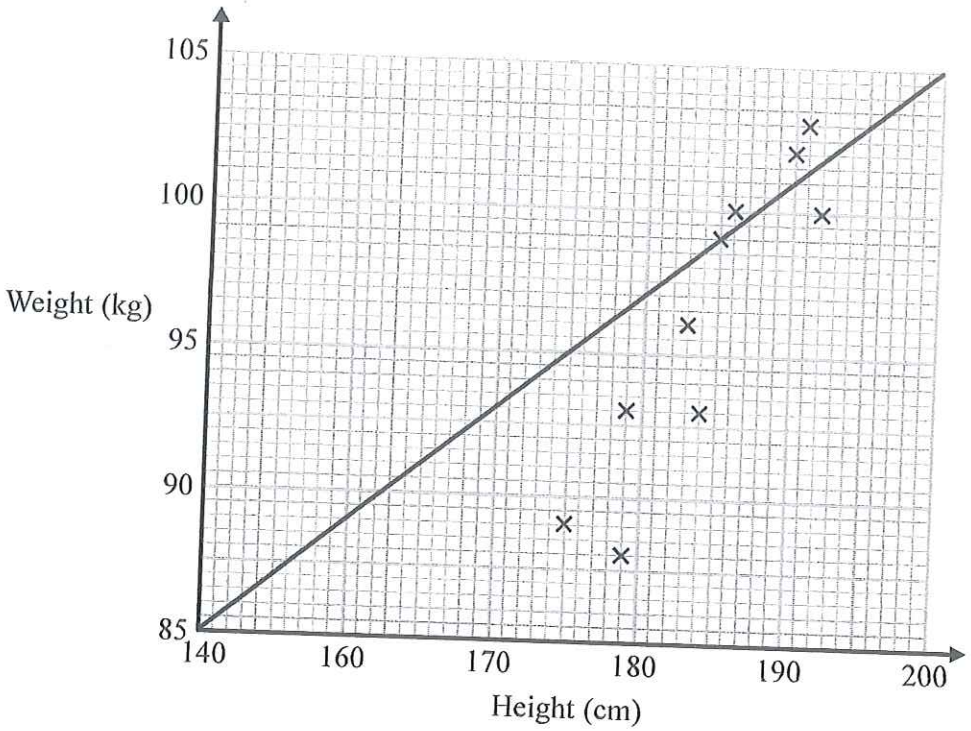
2 Sean has information about the height, in cm, and the weight, in kg, of each of ten rugby players.

He is asked to draw a scatter graph and a line of best fit for this information.

Here is his answer.



Sean has plotted the points accurately.

Write down two things that are wrong with his answer.

1 The scale on the height axis is not consistent. It goes up by 20 from 140 to 160 then goes up in 10s.

2 The line of best fit does not follow the trend of the data.

(Total for Question 2 is 2 marks)

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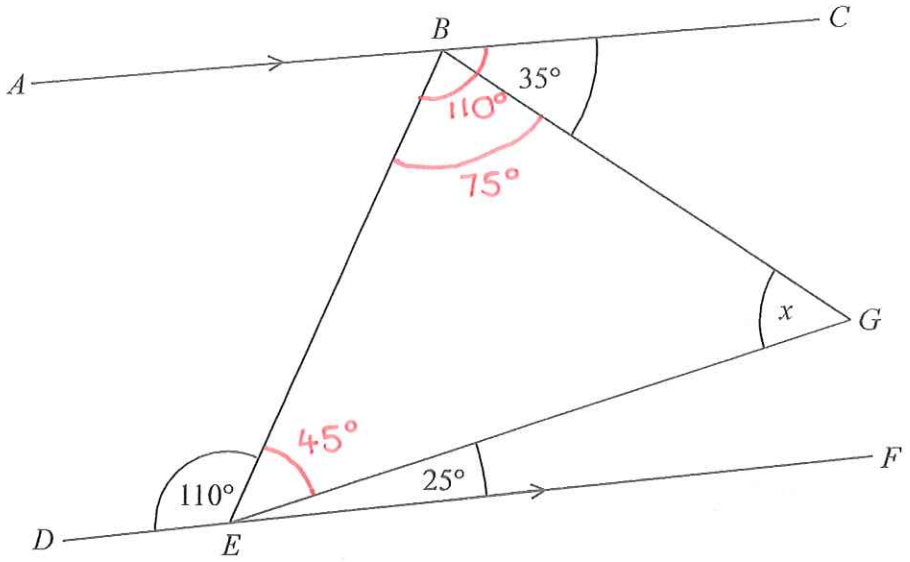
3 *BEG* is a triangle.



ABC and *DEF* are parallel lines.

Work out the size of angle *x*.
Give a reason for each stage of your working.

Angle CBE = 110° (alternate angles are equal.)

Angle GBE = 110 - 35 = 75°

Angle BEG = 180 - 110 - 25 = 45° (angles on a straight line add up to 180°.)

x = 180 - 75 - 45 (angles in triangle BEG add up to 180°.)

= 60°

60 °

(Total for Question 3 is 4 marks)

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4 Northern Bank has two types of account.
Both accounts pay compound interest.

Cash savings account
Interest
2.5% per annum

Shares account
Interest
3.5% per annum

Ali invests £2000 in the cash savings account.
Ben invests £1600 in the shares account.

(a) Work out who will get the most interest by the end of 3 years.
You must show all your working.

Ali

After 3 years:

$£2000 \times 1.025^3 = £2153.78$

Interest = £153.78

Ben

After 3 years:

$£1600 \times 1.035^3 = £1773.95$

Interest = £173.95

Ben receives the most interest by the end of 3 years.

(4)

In the 3rd year the rate of interest for the shares account is changed to 4% per annum.

(b) Does this affect who will get the most interest by the end of 3 years?
Give a reason for your answer.

No. This will increase the amount of interest Ben receives but he already receives more than Ali.

(1)

(Total for Question 4 is 5 marks)

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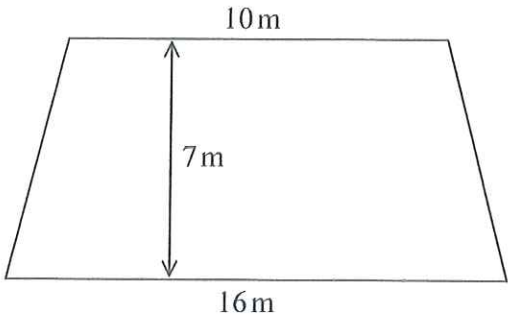
5 The diagram shows a floor in the shape of a trapezium.



£16.99

John is going to paint the floor.

Each 5 litre tin of paint costs £16.99
1 litre of paint covers an area of $2\,m^2$

John has £160 to spend on paint.

Has John got enough money to buy all the paint he needs?
You must show how you get your answer.

Area of floor = $\frac{1}{2}(16+10) \times 7$

$= 91\,m^2$

one tin covers $10m^2$

$91 \div 10 = 9.1$. John needs 10 tins.

Cost = £16.99 × 10 = £169.90

No, John does not have enough money.

(Total for Question 5 is 5 marks)

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6 A is the point with coordinates (5, 9)
B is the point with coordinates (d, 15)

The gradient of the line AB is 3

Work out the value of d.

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$$3 = \frac{15-9}{d-5}$$

$$= \frac{6}{d-5}$$

$$d-5 = 2$$

$$d = 7$$

7

(Total for Question 6 is 3 marks)

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7 (a) Write the number 0.000 086 23 in standard form.

8.623×10^{-5}

(1)

(b) Work out $\dfrac{3.2 \times 10^{3} + 5.1 \times 10^{-2}}{4.3 \times 10^{-4}}$

Give your answer in standard form, correct to 3 significant figures.

$\dfrac{3.2 \times 10^{3} + 5.1 \times 10^{-2}}{4.3 \times 10^{-4}} = 7,441,979.07$

$= 7.44 \times 10^{6}$ (3s.f.)

7.44×10^{6}

(2)

(Total for Question 7 is 3 marks)

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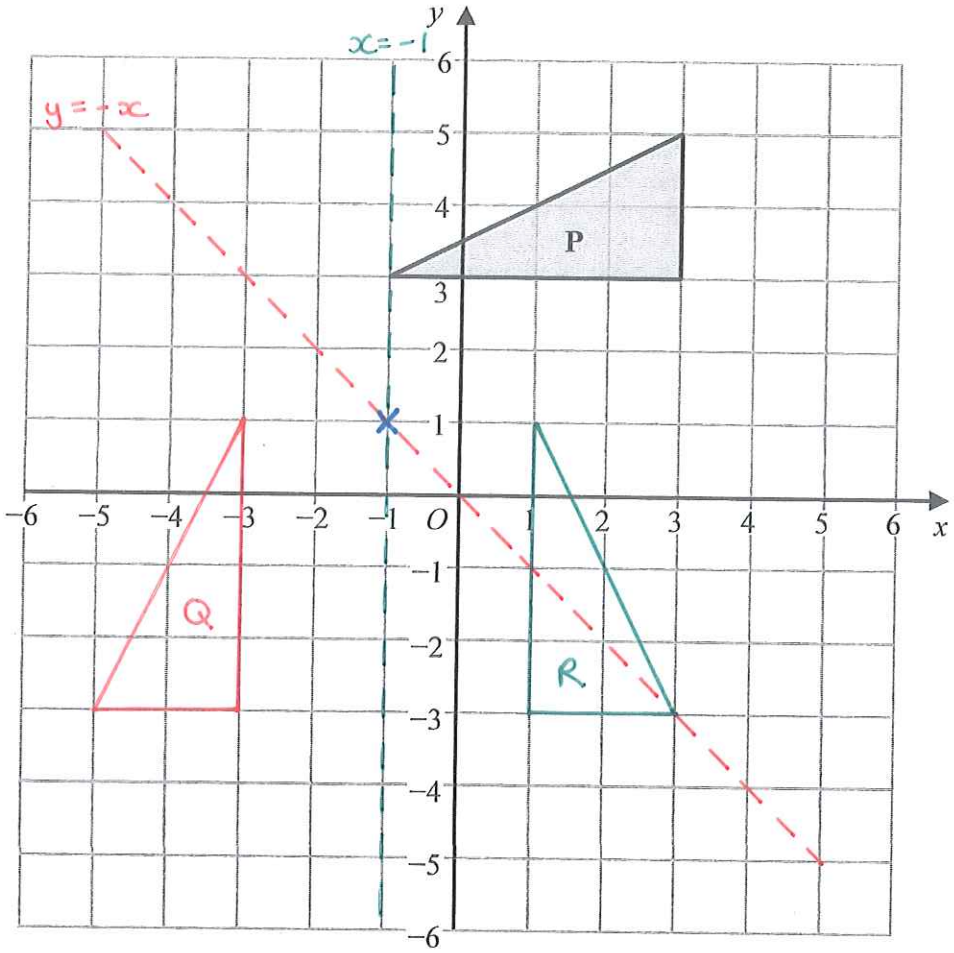

Triangle **P** is reflected in the line $y = -x$ to give triangle **Q**.
Triangle **Q** is reflected in the line $x = -1$ to give triangle **R**.

Describe fully the single transformation that maps triangle **R** to triangle **P**.

Rotation 90° anticlockwise about (−1, 1).

(Total for Question 8 is 3 marks)

9 Martin truncates the number N to 1 digit.
The result is 7

Write down the error interval for N.

$7 \leq N < 8$

(Total for Question 9 is 2 marks)

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10 Robert makes 50 litres of green paint by mixing litres of yellow paint and litres of blue paint in the ratio 2 : 3

Yellow paint is sold in 5 litre tins.
Each tin of yellow paint costs £26

Blue paint is sold in 10 litre tins.
Each tin of blue paint costs £48

Robert sells all the green paint he makes in 10 litre tins.
He sells each tin of green paint for £66.96

Work out Robert's percentage profit on each tin of green paint he sells.

Green paint : $50 \div 5 = 10$

$10 \times 2 = 20$ litres of yellow

$10 \times 3 = 30$ litres of blue.

Yellow : $4 \times £26 = £104$

Blue : $3 \times £48 = £144$

Total spend = £248

spend per tin of green = $£248 \div 5$

$= £49.60$

profit per tin of green = $£66.96 - £49.60$

$= £17.36$

percentage profit per tin = $\frac{17.36}{49.60} \times 100$

$= 35\%$

35 %

(Total for Question 10 is 5 marks)

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11 In a restaurant there are

9 starter dishes
15 main dishes
8 dessert dishes

Janet is going to choose one of the following combinations for her meal.

a starter dish and a main dish
or a main dish and a dessert dish
or a starter dish, a main dish and a dessert dish

Show that there are 1335 different ways to choose the meal.

Starter and main: $9 \times 15 = 135$

main and dessert: $15 \times 8 = 120$

Starter, main and dessert: $9 \times 15 \times 8 = 1080$

Total $= 1080 + 120 + 135$

$= 1335$ different ways.

(Total for Question 11 is 3 marks)

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12 (a) Write $\frac{4x^2 - 9}{6x + 9} \times \frac{2x}{x^2 - 3x}$ in the form $\frac{ax + b}{cx + d}$ where a, b, c and d are integers.

$$\frac{4x^2 - 9}{6x + 9} \times \frac{2x}{x^2 - 3x}$$

$$= \frac{(2x + 3)(2x - 3)}{3(2x + 3)} \times \frac{2x}{x(x - 3)}$$

$$= \frac{2(2x - 3)}{3(x - 3)}$$

$$= \frac{4x - 6}{3x - 9}$$

$\frac{4x-6}{3x-9}$

(3)

(b) Express $\frac{3}{x+1} + \frac{1}{x-2} - \frac{4}{x}$ as a single fraction in its simplest form.

$$\frac{3}{x+1} + \frac{1}{x-2} - \frac{4}{x} = \frac{3x(x-2) + x(x+1) - 4(x+1)(x-2)}{x(x+1)(x-2)}$$

$$= \frac{3x^2 - 6x + x^2 + x - 4(x^2 - x - 2)}{x(x+1)(x-2)}$$

$$= \frac{3x^2 - 6x + x^2 + x - 4x^2 + 4x + 8}{x(x+1)(x-2)}$$

$$= \frac{-x + 8}{x(x+1)(x-2)}$$

$\frac{-x+8}{x(x+1)(x-2)}$

(3)

(Total for Question 12 is 6 marks)

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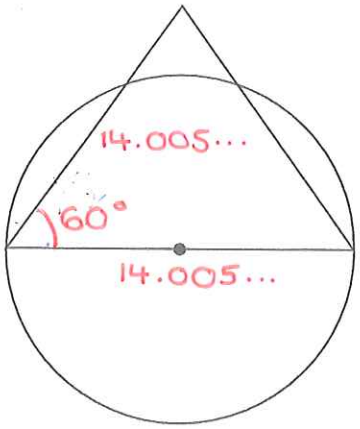
13 The diagram shows a circle and an equilateral triangle.

One side of the equilateral triangle is a diameter of the circle.
The circle has a circumference of 44 cm.

Work out the area of the triangle.
Give your answer correct to 3 significant figures.



circumference = $\pi \times d$

$44 = \pi \times d$

$d = 44 \div \pi = 14.00563\ldots$

Area of triangle = $\frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 14.005\ldots \times 14.005\ldots \times \sin 60°$

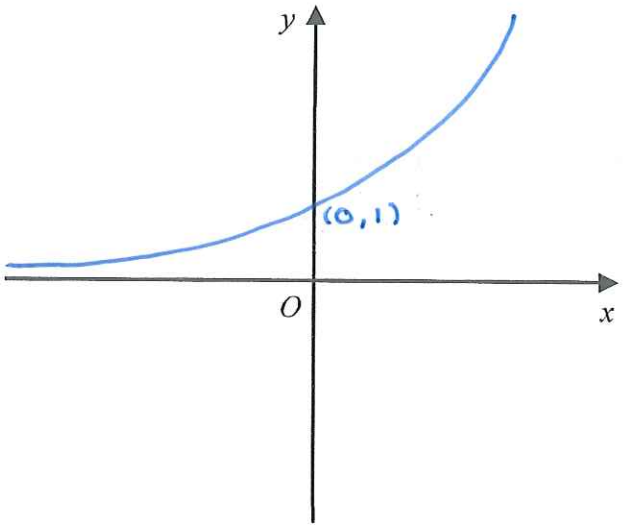
$= 84.9388\ldots$

$= 84.9$ cm² (3s.f.)

84.9 ..... cm²

(Total for Question 13 is 3 marks)

14 On the grid, sketch the curve with equation $y = 2^x$
Give the coordinates of any points of intersection with the axes.



(Total for Question 14 is 2 marks)

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15 The equation of a circle is $x^2 + y^2 = 42.25$

Find the radius of the circle.

$\sqrt{42.25} = 6.5$

6.5

(Total for Question 15 is 1 mark)

16 There are only red counters and blue counters in a bag.

Joe takes at random a counter from the bag.
The probability that the counter is red is 0.65
Joe puts the counter back into the bag.

Mary takes at random a counter from the bag.
She puts the counter back into the bag.

(a) What is the probability that Joe and Mary take counters of different colours?

P(Joe red and Mary blue) = $0.65 \times 0.35 = 0.2275$

P(Joe blue and Mary red) = $0.35 \times 0.65 = 0.2275$

$0.2275 + 0.2275 = 0.455$

0.455

(2)

There are 78 red counters in the bag.

(b) How many blue counters are there in the bag?

$\frac{78}{\text{total}} = 0.65$

$\text{total} = \frac{78}{0.65} = 120$

blue = $120 - 78 = 42$

42

(2)

(Total for Question 16 is 4 marks)

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17 p and q are two numbers such that $p > q$

When you subtract 5 from p and subtract 5 from q the answers are in the ratio 5 : 1
When you add 20 to p and add 20 to q the answers are in the ratio 5 : 2

Find the ratio $p : q$
Give your answer in its simplest form.

$p - 5 : q - 5$

$5 : 1$

$\frac{p-5}{5} = \frac{q-5}{1}$ ①

$p + 20 : q + 20$

$5 : 2$

$\frac{p+20}{5} = \frac{q+20}{2}$ ②

① : $p - 5 = 5(q - 5)$

$p - 5 = 5q - 25$

$-5q]$ $p - 5q - 5 = -25$

$+5]$ $p - 5q = -20$ ③

② : $2(p + 20) = 5(q + 20)$

$2p + 40 = 5q + 100$

$-5q]$ $2p - 5q + 40 = 100$

$-40]$ $2p - 5q = 60$ ④

④ − ③ :

$2p - 5q = 60$

$p - 5q = -20$ −

$p = 80$

substitute into ③ : $80 - 5q = -20$

$5q = 100$

$q = 20$

$p : q$

$80 : 20$

$4 : 1$

4 : 1

(Total for Question 17 is 5 marks)

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18 The straight line L_1 passes through the points with coordinates (4, 6) and (12, 2)
The straight line L_2 passes through the origin and has gradient -3

The lines L_1 and L_2 intersect at point P.

Find the coordinates of P.

L_1 : gradient $= \frac{2-6}{12-4} = -\frac{1}{2}$ $\qquad$ L_2 : $y = -3x$ ②

$y - y_1 = -\frac{1}{2}(x - x_1)$

$y - 6 = -\frac{1}{2}(x - 4)$

$= -\frac{1}{2}x + 2$

$y = -\frac{1}{2}x + 8$ ①

equate ① and ② : $-\frac{1}{2}x + 8 = -3x$

$+3x]$ $\frac{5}{2}x + 8 = 0$

$-8]$ $\frac{5}{2}x = -8$

$\times \frac{2}{5}]$ $x = -3.2$

Substitute into ② : $y = -3 \times -3.2$

$= 9.6$

(-3.2 , 9.6)

(Total for Question 18 is 4 marks)

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19 Solve $22 < \frac{m^2 + 7}{4} < 32$

Show all your working.

$22 < \frac{m^2+7}{4} < 32$

$\times 4]$ $88 < m^2 + 7 < 128$

$-7]$ $81 < m^2 < 121$

$\pm\sqrt{\ }]$ $-11 < m < -9$ or $9 < m < 11$

$-11 < m < -9$ or $9 < m < 11$

(Total for Question 19 is 5 marks)

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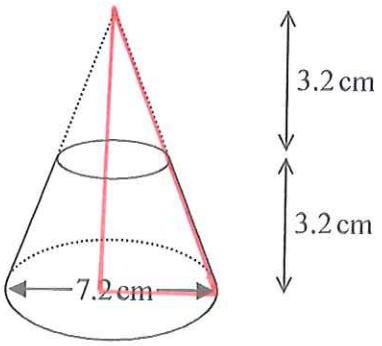
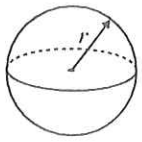
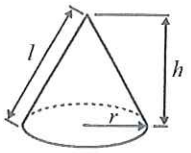
20 Here is a frustum of a cone.



Volume of sphere $= \frac{4}{3}\pi r^3$



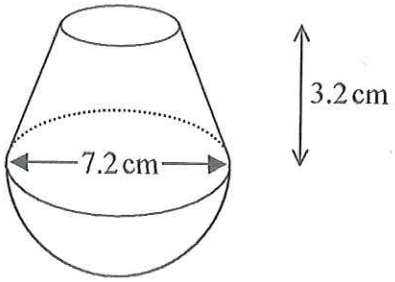
Volume of cone $= \frac{1}{3}\pi r^2 h$



The diagram shows that the frustum is made by removing a cone with height 3.2 cm from a solid cone with height 6.4 cm and base diameter 7.2 cm.

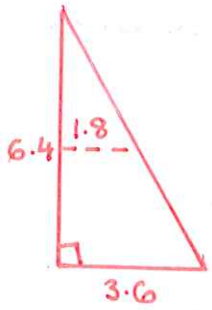
The frustum is joined to a solid hemisphere of diameter 7.2 cm to form the solid S shown below.

Solid S



The density of the frustum is 2.4 g/cm^3
The density of the hemisphere is 4.8 g/cm^3

Calculate the average density of solid S.



Volume of frustum $= \frac{1}{3}\pi \times 3.6^2 \times 6.4 - \frac{1}{3}\pi \times 1.8^2 \times 3.2$

$= \frac{3024\pi}{125} = 76.00 \text{ cm}^3$ (2dp)

Mass = Volume x Density

Mass of frustum = 76.00 x 2.4

= 182.40 g (2dp)

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Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3} \times \pi \times 3.6^3$

$= \frac{3888\pi}{125} = 97.72 \text{ cm}^3 \text{ (2dp)}$

Mass of hemisphere $= 97.72 \times 4.8$

$= 469.04 \text{ g (2dp)}$

Total mass $= 469.04 + 182.40 = 651.44 \text{ g (2dp)}$

Total volume $= 76.00 + 97.72 = 173.72 \text{ cm}^3 \text{ (2dp)}$

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

Arerage density of $S = \frac{651.44}{173.72}$

$= 3.75 \text{ g/cm}^3 \text{ (3s.f.)}$

3.75 g/cm³

(Total for Question 20 is 5 marks)

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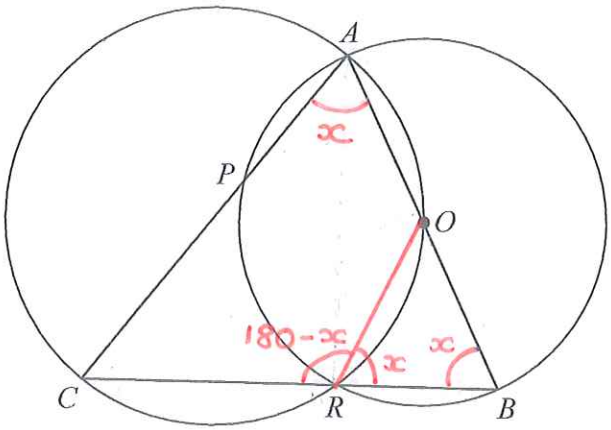

A, B, R and P are four points on a circle with centre O.
A, O, R and C are four points on a different circle.
The two circles intersect at the points A and R.

CPA, CRB and AOB are straight lines.

Prove that angle CAB = angle ABC.

Let angle CAB = x

then angle ORC = $180 - x$ (opposite angles in a cyclic quadrilateral add to 180°)

angle ORB = x (angles on a straight line add to 180°)

angle ABC = x (OR = OB as they are equal radii and base angles of an isosceles triangle are equal.)

∴ angle CAB = angle ABC.

(Total for Question 21 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS

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