

1 hour 30 mins

## LPGS GCSE March Mock Paper 3

## HIGHER TIER

Thursday 14<sup>th</sup> March 2019 MORNINGName KGMaths Teacher WORKED SOLUTIONS

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may be used.**
- If your calculator does not have a  $\pi$  button, take the value of  $\pi$  to be 3.142 unless the question instructs otherwise.



## Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

TOTAL MARK out 80 \_\_\_\_\_

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 (a) Write 7357 correct to 3 significant figures.

7360

(1)

- (b) Work out  $\frac{\sqrt{17 + 4^2}}{7.3^2}$

Write down all the figures on your calculator display.

0.107798135

(2)

(Total for Question 1 is 3 marks)

- 2 Last year Jo paid £245 for her car insurance.  
This year she has to pay £883 for her car insurance.

Work out the percentage increase in the cost of her car insurance.

$$\text{increase} = 883 - 245 = 638$$

$$\text{percentage increase} = \frac{638}{245} \times 100 = 260.408\dots$$

$$= 260\% \text{ (3s.f.)}$$

260

%

(Total for Question 2 is 3 marks)

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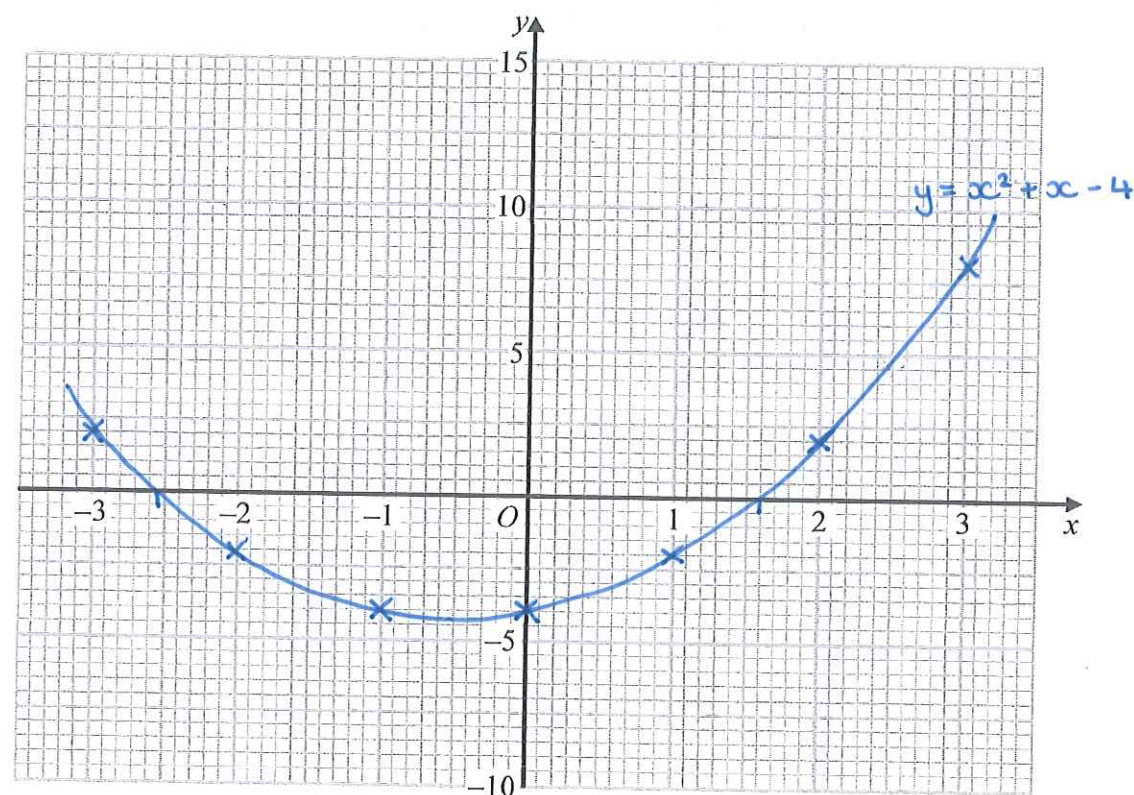


3 (a) Complete this table of values for  $y = x^2 + x - 4$

x	-3	-2	-1	0	1	2	3
y	2	-2	-4	-4	-2	2	8

(2)

(b) On the grid, draw the graph of  $y = x^2 + x - 4$  for values of  $x$  from -3 to 3



(2)

(c) Use the graph to estimate a solution to  $x^2 + x - 4 = 0$

1.6

(1)

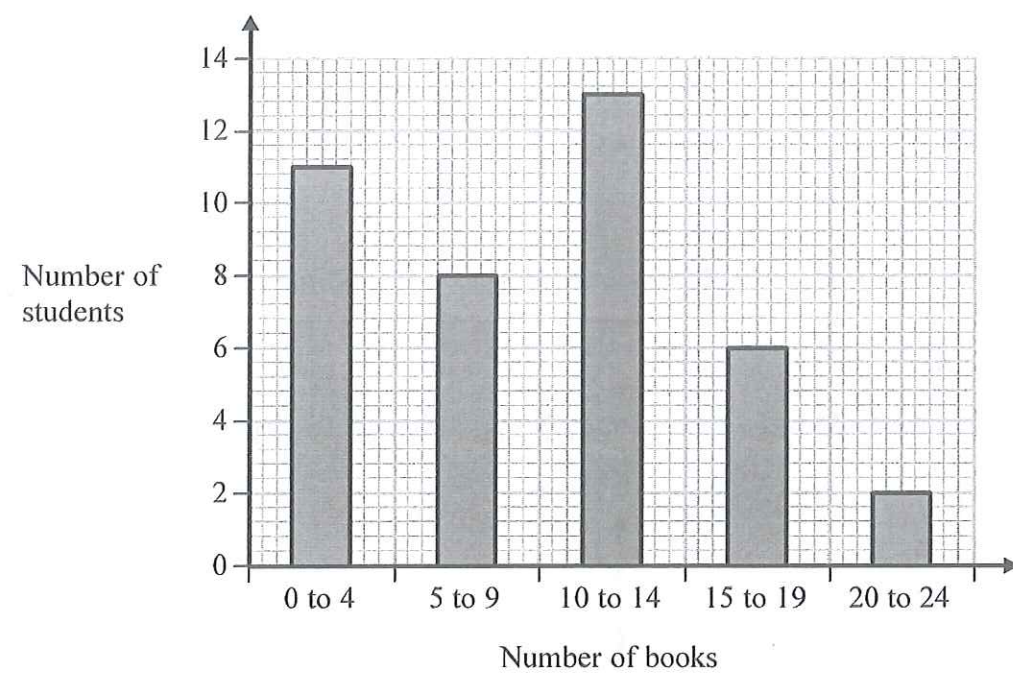
(Total for Question 3 is 5 marks)





- 4 Fran asks each of 40 students how many books they bought last year.

The chart below shows information about the number of books bought by each of the 40 students.



- (a) Work out the percentage of these students who bought 20 or more books.

$$\frac{2}{40} \times 100 = 5$$

5 %  
(2)

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- (b) Show that an estimate for the mean number of books bought is 9.5  
You must show all your working.

Number of books	Frequency, $f$	midpoint, $x$	$f \times x$
0-4	11	2	22
5-9	8	7	56
10-14	13	12	156
15-19	6	17	102
20-24	2	22	44
Total :	40		380

$$\text{Mean} = \frac{380}{40} = 9.5$$

(4)

(Total for Question 4 is 6 marks)

5 Lara is a skier.

She completed a ski race in 1 minute 54 seconds.  
The race was 475 m in length.

Lara assumes that her average speed is the same for each race.

- (a) Using this assumption, work out how long Lara should take to complete a 700 m race.  
Give your answer in minutes and seconds.

$$\begin{aligned}\text{Speed} &= \frac{\text{Distance}}{\text{Time}} \\ &= \frac{475}{114} \\ &= 4\frac{1}{6} \text{ metres per second}\end{aligned}$$

$$\begin{aligned}\text{Time} &= \frac{\text{Distance}}{\text{Speed}} \\ &= \frac{700}{4\frac{1}{6}} \\ &= 168 \text{ seconds} \\ &= 2 \text{ minutes } 48 \text{ seconds}\end{aligned}$$

2 minutes 48 seconds  
(3)

Lara's average speed actually increases the further she goes.

- (b) How does this affect your answer to part (a)?

My answer to (a) will be too high. She will complete the 700m race in less than 2 minutes 48 seconds.

(1)

(Total for Question 5 is 4 marks)

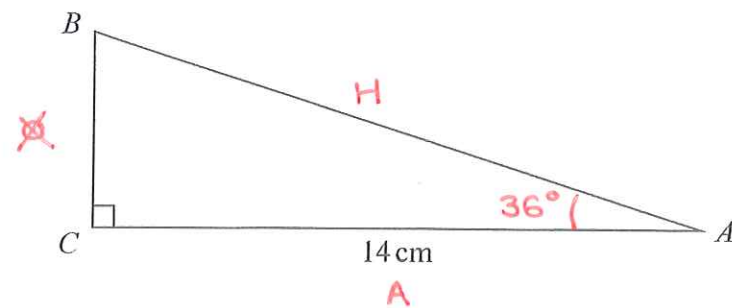
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6  $ABC$  is a right-angled triangle.



$AC = 14$  cm.  
Angle  $C = 90^\circ$

size of angle  $B$  : size of angle  $A = 3 : 2$

Work out the length of  $AB$ .  
Give your answer correct to 3 significant figures.

$$A + B = 90^\circ$$

$$90 \div 5 = 18 \quad A = 18 \times 2 = 36^\circ$$

$$B = 18 \times 3 = 54^\circ$$



$$H = \frac{A}{C}$$

$$AB = \frac{14}{\cos 36}$$

$$= 17.3049 \dots$$

$$= 17.3 \text{ cm (3s.f.)}$$

17.3 cm

(Total for Question 6 is 4 marks)





7 The table gives information about the speeds of 70 cars.

Speed ( $s$ mph)	Frequency
$0 < s \leq 10$	14
$10 < s \leq 20$	18
$20 < s \leq 30$	26
$30 < s \leq 40$	12

Midpoint

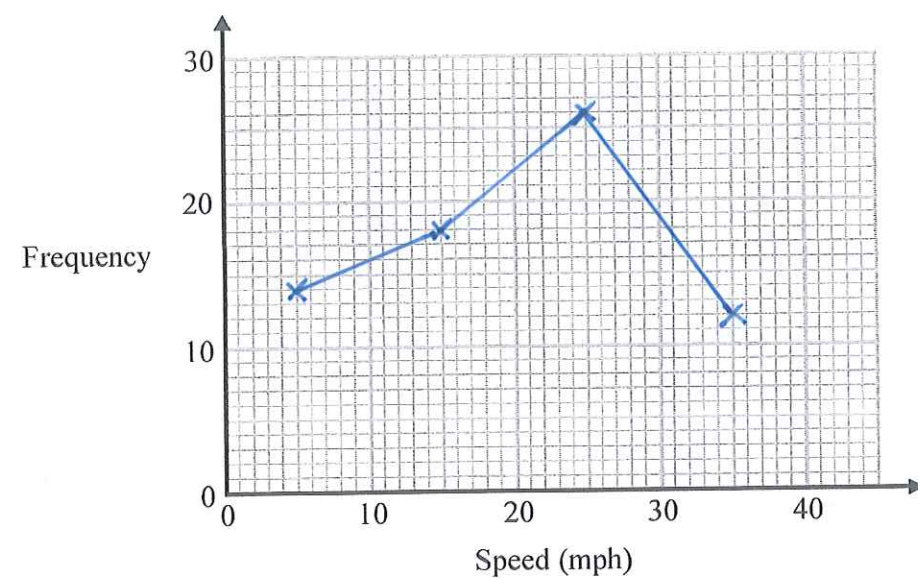
5

15

25

35

Draw a frequency polygon for this information.



(Total for Question 7 is 2 marks)



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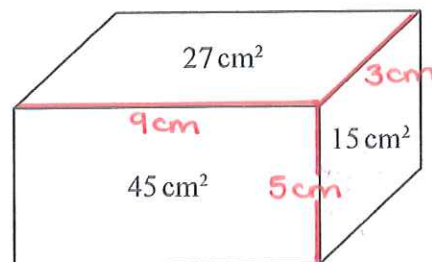
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- 8 The diagram shows a solid metal cuboid.

The areas of three of the faces are marked on the diagram.  
The lengths, in cm, of the edges of the cuboid are whole numbers.



The metal cuboid is melted and made into cubes.  
Each of the cubes has sides of length  $2.5\text{ cm}$ .

Work out the greatest number of these cubes that can be made.

$$\text{Volume of cuboid} = 9 \times 5 \times 3 = 135\text{ cm}^3$$

$$\text{Volume of one cube} = 2.5^3 = 15.625\text{ cm}^3$$

$$135 \div 15.625 = 8.64$$

8 cubes can be made.

8

(Total for Question 8 is 5 marks)

9 (a) Expand and simplify  $(x-2)(2x+3)(x+1)$

$$\begin{aligned} &= (x-2)(2x^2+3x+2x+3) \\ &= (x-2)(2x^2+5x+3) \\ &= 2x^3+5x^2+3x-4x^2-10x-6 \\ &= 2x^3+x^2-7x-6 \end{aligned}$$

$$\frac{2x^3+x^2-7x-6}{(3)}$$

$$\frac{y^4 \times y^n}{y^2} = y^{-3}$$

(b) Find the value of  $n$ .

$$\begin{aligned} \frac{y^4 \times y^n}{y^2} &= y^{-3} \\ y^{4+n-2} &= y^{-3} \\ 2+n &= -3 \\ n &= -5 \end{aligned}$$

$$\frac{-5}{(2)}$$

(c) Solve  $5x^2 - 4x - 3 = 0$

Give your solutions correct to 3 significant figures.

$$5x^2 - 4x - 3 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 5 \times -3}}{10}$$

$$= 1.27177... \text{ or } -0.47177...$$

$$= 1.27 \text{ or } -0.472 \text{ (3s.f.)}$$

$$\frac{1.27 \text{ or } -0.472}{(3)}$$

(Total for Question 9 is 8 marks)

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10  $f(x) = 4\sin x^\circ$

(a) Find  $f(23)$

Give your answer correct to 3 significant figures.

$$4\sin 23^\circ = 1.56292\dots$$

$$= 1.56 \text{ (3s.f.)}$$

$$1.56$$

(1)

$g(x) = 2x - 3$

(b) Find  $fg(34)$

Give your answer correct to 3 significant figures.

$$g(34) = 2 \times 34 - 3$$

$$= 65$$

$$f(65) = 4\sin 65$$

$$= 3.62523\dots$$

$$= 3.63 \text{ (3s.f.)}$$

$$3.63$$

(2)

$h(x) = (x + 4)^2$

Ivan needs to solve the following equation  $h(x) = 25$

He writes

$$(x + 4)^2 = 25$$

$$x + 4 = 5$$

$$x = 1$$

This is not fully correct.

(c) Explain why.

$$(x + 4)^2 = 25 \text{ gives } x + 4 = 5 \text{ or } x + 4 = -5.$$

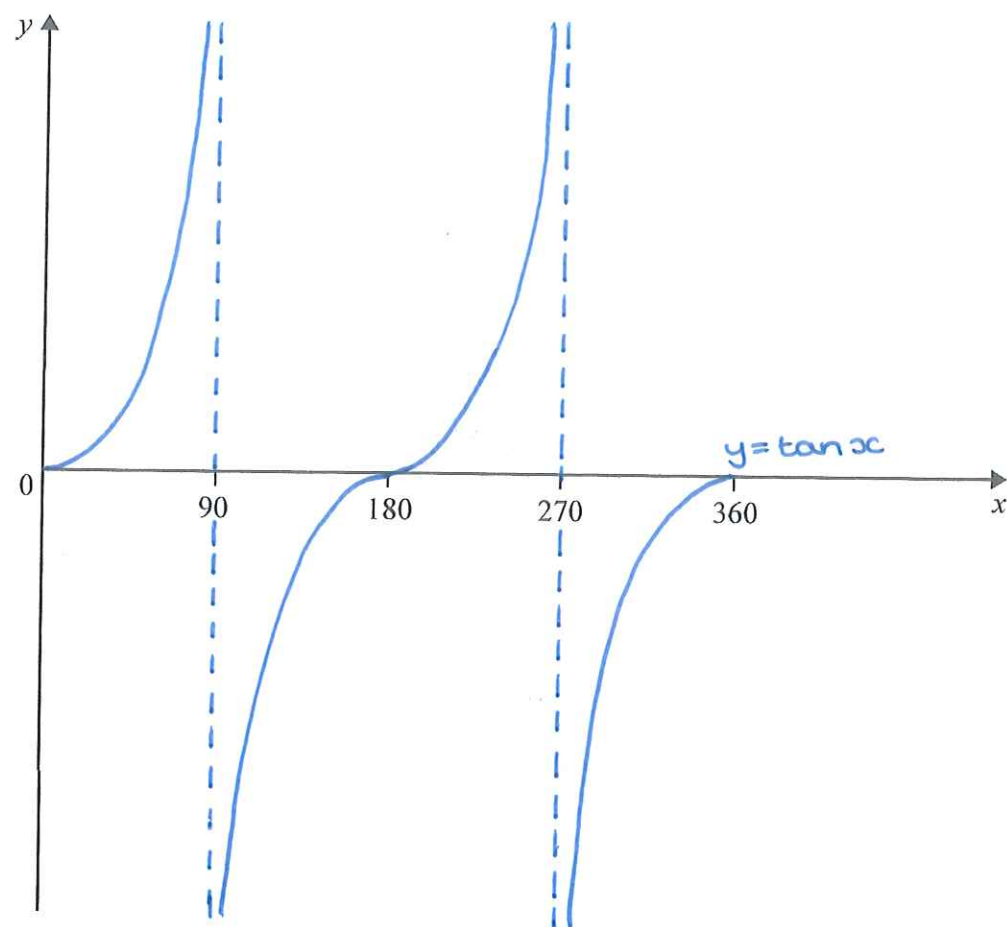
(1)

(Total for Question 10 is 4 marks)





11 Sketch the graph of  $y = \tan x^\circ$  for  $0 \leq x \leq 360$



(Total for Question 11 is 2 marks)

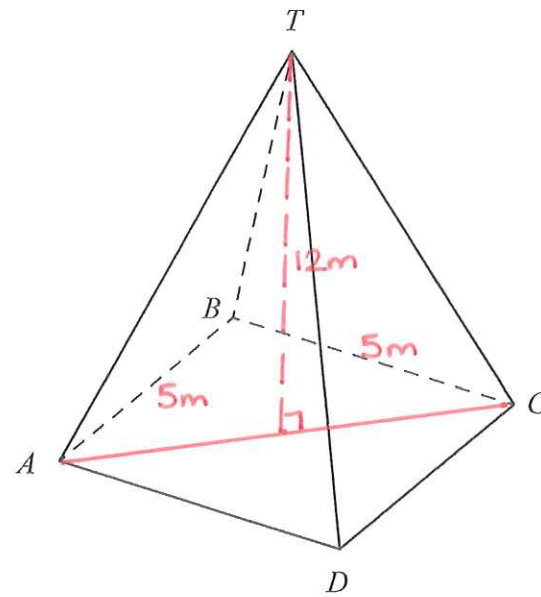
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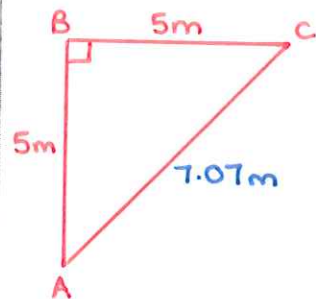
12 Here is a pyramid with a square base  $ABCD$ .



$$AB = 5 \text{ m}$$

The vertex  $T$  is 12 m vertically above the midpoint of  $AC$ .

Calculate the size of angle  $TAC$ .



$$AC^2 = 5^2 + 5^2$$

$$= 50$$

$$AC = \sqrt{50} = 7.0710 \dots \text{ m}$$

$$7.0710 \dots \div 2 = 3.5355 \dots \text{ m}$$



$$\tan \hat{TAC} = \frac{12}{3.535 \dots}$$

$$\hat{TAC} = \tan^{-1} \left( \frac{12}{3.535 \dots} \right) = 73.5835 \dots$$

$$= 73.6^\circ \text{ (1dp)}$$

$$73.6^\circ$$

(Total for Question 12 is 4 marks)

- 13 The number of animals in a population at the start of year  $t$  is  $P_t$ .  
The number of animals at the start of year 1 is 400

Given that

$$P_{t+1} = 1.01P_t$$

work out the number of animals at the start of year 3

$$P_1 = 400$$

$$P_2 = 1.01 \times 400 = 404$$

$$P_3 = 1.01 \times 404 = 408.4$$

408

(Total for Question 13 is 2 marks)

- 14  $y$  is inversely proportional to  $x^3$

$$y = 44 \text{ when } x = a$$

Show that  $y = 5.5$  when  $x = 2a$

$$y \propto \frac{1}{x^3}$$

$$y = \frac{k}{x^3}$$

$$y = 44, x = a: 44 = \frac{k}{a^3}$$

$$k = 44a^3$$

$$y = \frac{44a^3}{x^3}$$

$$\begin{aligned} x = 2a: y &= \frac{44a^3}{(2a)^3} \\ &= \frac{44a^3}{8a^3} = \frac{44}{8} = 5.5 \end{aligned}$$

(Total for Question 14 is 3 marks)

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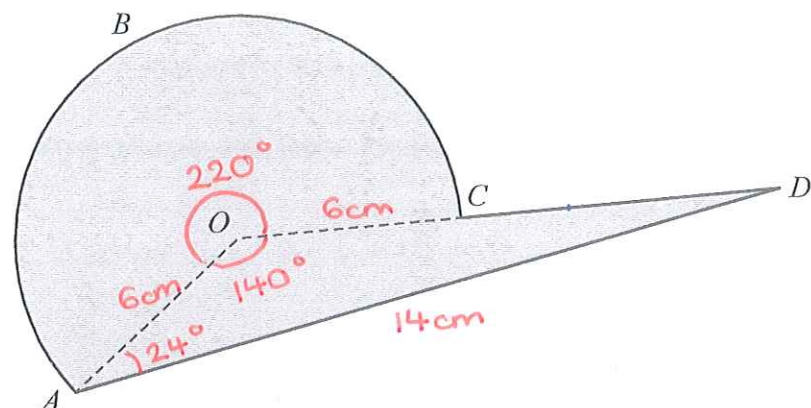
- 15 Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

$$\begin{aligned}(2n+3)^2 - (2n+1)^2 &= (2n+3)(2n+3) - (2n+1)(2n+1) \\&= 4n^2 + 6n + 6n + 9 - (4n^2 + 2n + 2n + 1) \\&= 4n^2 + 12n + 9 - (4n^2 + 4n + 1) \\&= 8n + 8 \\&= 8(n+1) \text{ which is a multiple} \\&\quad \text{of 8.}\end{aligned}$$

$\therefore$  the difference between the squares of any two consecutive odd numbers is always a multiple of 8.

(Total for Question 15 is 3 marks)

16 Here is a shaded shape  $ABCD$ .



The shape is made from a triangle and a sector of a circle, centre  $O$  and radius 6 cm.  
 $OCD$  is a straight line.

$AD = 14$  cm  
 Angle  $AOD = 140^\circ$   
 Angle  $OAD = 24^\circ$

Calculate the perimeter of the shape.  
 Give your answer correct to 3 significant figures.

$$\text{Arc length } ABC = \frac{220}{360} \times \pi \times 12 = \frac{22}{3} \pi = 23.03 \text{ cm (2dp)}$$

Cosine rule:

$$\begin{aligned} OD^2 &= 6^2 + 14^2 - 2 \times 6 \times 14 \cos 24^\circ \\ &= 78.5243 \dots \end{aligned}$$

$$OD = 8.86139 \dots = 8.86 \text{ cm (2dp)}$$

$$CD = 8.86 - 6 = 2.86 \text{ cm (2dp)}$$

$$\begin{aligned} \text{Total perimeter} &= 23.03 + 2.86 + 14 \\ &= 39.9 \text{ cm (3s.f.)} \end{aligned}$$

39.9 cm

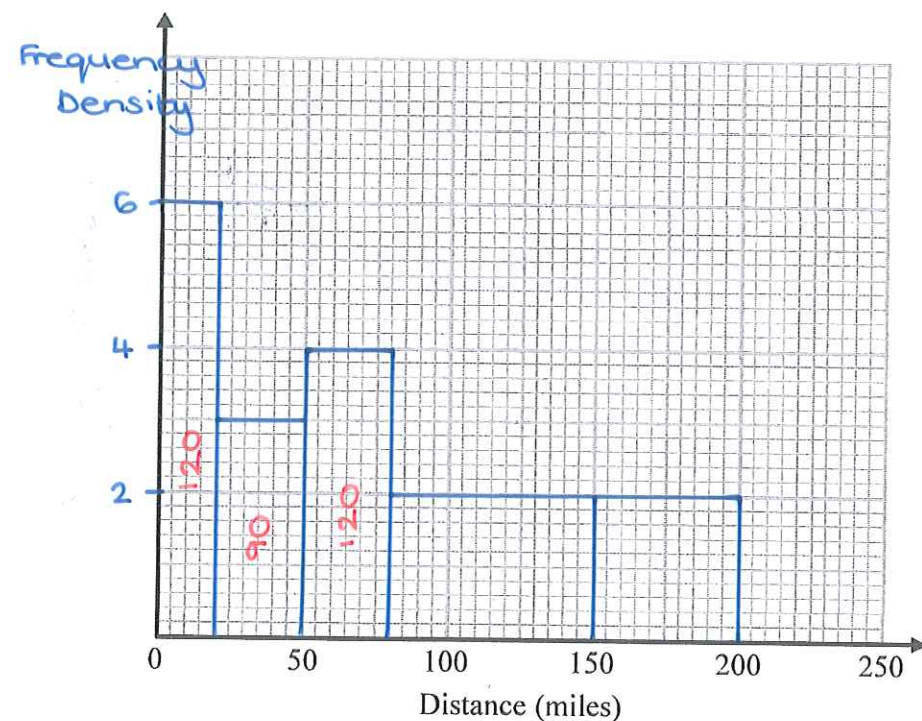
(Total for Question 16 is 5 marks)



17 The table shows information about the distances 570 students travelled to a university open day.

Distance ( $d$ miles)	Frequency	Group width	Frequency density
$0 < d \leq 20$	120	20	6
$20 < d \leq 50$	90	30	3
$50 < d \leq 80$	120	30	4
$80 < d \leq 150$	140	70	2
$150 < d \leq 200$	100	50	2

(a) Draw a histogram for the information in the table.



(3)

(b) Estimate the median distance.

$$\frac{570}{2} = 285$$

$$\frac{75}{120} \times 30 = 18.75$$

$$50 + 18.75 = 68.75$$

68.75 miles

(2)

(Total for Question 17 is 5 marks)





18 A high speed train travels a distance of 487 km in 3 hours.

The distance is measured correct to the nearest kilometre.

The time is measured correct to the nearest minute.

By considering bounds, work out the average speed, in km/minute, of the train to a suitable degree of accuracy.

You must show all your working and give a reason for your answer.

Distance:

$$\text{upper bound} = 487.5 \text{ km}$$

$$\text{lower bound} = 486.5 \text{ km}$$

Time:

$$\text{upper bound} = 180.5 \text{ minutes}$$

$$\text{lower bound} = 179.5 \text{ minutes}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed: upper bound} = \frac{487.5}{179.5} = 2.71587... \text{ km/minute}$$

$$\text{lower bound} = \frac{486.5}{180.5} = 2.69529... \text{ km/minute}$$

$$2.69529... < \text{speed} < 2.71587...$$

$$\text{speed} = 2.7 \text{ km/minute (1dp)}$$

as both 2.695... and 2.715... round to 2.7.

$$\dots\dots\dots 2.7 \dots\dots\dots \text{ km/minute}$$

(Total for Question 18 is 5 marks)

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19 Solve algebraically the simultaneous equations

$$2x^2 - y^2 = 17 \quad (1)$$

$$x + 2y = 1 \quad (2)$$

$$(2): x = 1 - 2y \quad (3)$$

Substitute (3) into (1)  $2(1-2y)^2 - y^2 = 17$

$$2(1 - 4y + 4y^2) - y^2 = 17$$

$$2 - 8y + 8y^2 - y^2 = 17$$

$$7y^2 - 8y - 15 = 0$$

$$(7y - 15)(y + 1) = 0$$

$$y = \frac{15}{7} \text{ or } y = -1$$

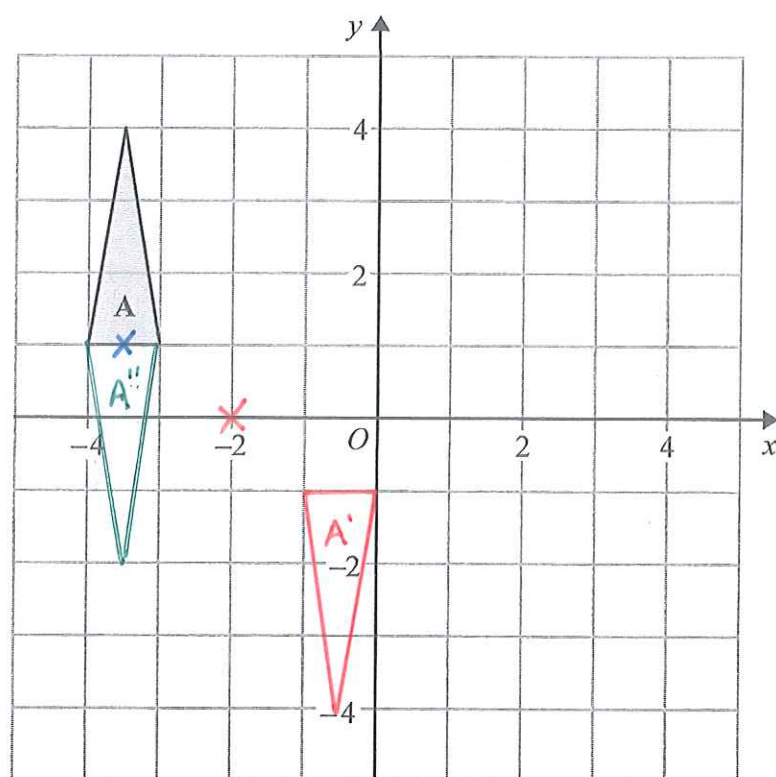
Substitute  $y = -1$  into (3):  $x = 1 - 2(-1)$   
 $= 3$

Substitute  $y = \frac{15}{7}$  into (3):  $x = 1 - 2 \times \frac{15}{7}$   
 $= -\frac{23}{7}$

$$x = 3 \text{ and } y = -1 \text{ or } x = -\frac{23}{7} \text{ and } y = \frac{15}{7}$$

(Total for Question 19 is 5 marks)





Triangle A is transformed by the combined transformation of a rotation of  $180^\circ$  about the point  $(-2, 0)$  followed by a translation with vector  $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

One point on triangle A is invariant under the combined transformation.

Find the coordinates of this point.

( -3.5 , 1 )

(Total for Question 20 is 2 marks)

TOTAL FOR PAPER IS 80 MARKS

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