LPGS GCSE Mock 2 Paper 1

HIGHER TIER

Tuesday 25th February 2020 MORNING

Name LJH

Maths Teacher Worked Solutions

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may not be used.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

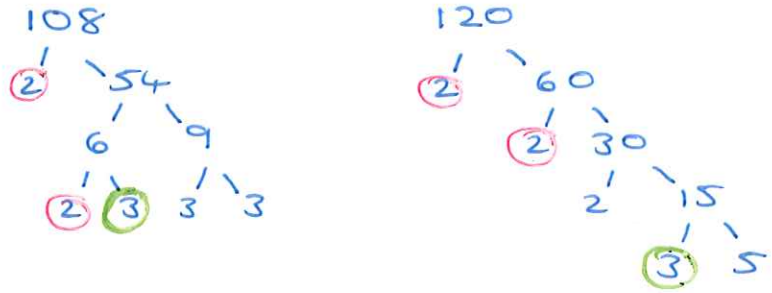
TOTAL MARK out 80 _____

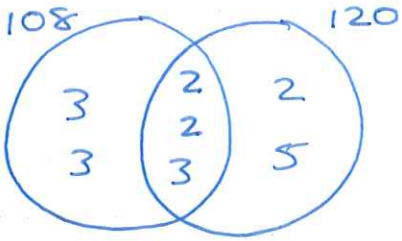
Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Find the Lowest Common Multiple (LCM) of 108 and 120





LCM = 3×3×2×2×3×2×5
= 9 × 12 × 10
= 108 × 10
= 1080

Alternative method

108, 216, 324, 432, 540, 648, 756, 864, 972, 1080, ...

120, 240, 360, 480, 600, 720, 840, 960, 1080, ...

1080

(Total for Question 1 is 3 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

2 There are 60 people in a choir.
Half of the people in the choir are women. 30 women

The number of women in the choir is 3 times the number of men in the choir. 10 men
The rest of the people in the choir are children. 20 children

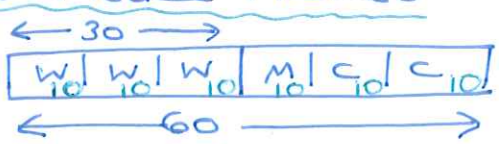
the number of children in the choir : the number of men in the choir = n : 1

Work out the value of n.
You must show how you get your answer.

children : men
20 : 10
2 : 1

$n = 2$

Bar model method



$n =$ 2

(Total for Question 2 is 4 marks)

3 Work out $1\frac{3}{4} \times 1\frac{1}{3}$

Give your answer as a mixed number.

$\frac{7}{4} \times \frac{4}{3}$

$= \frac{28}{12}$

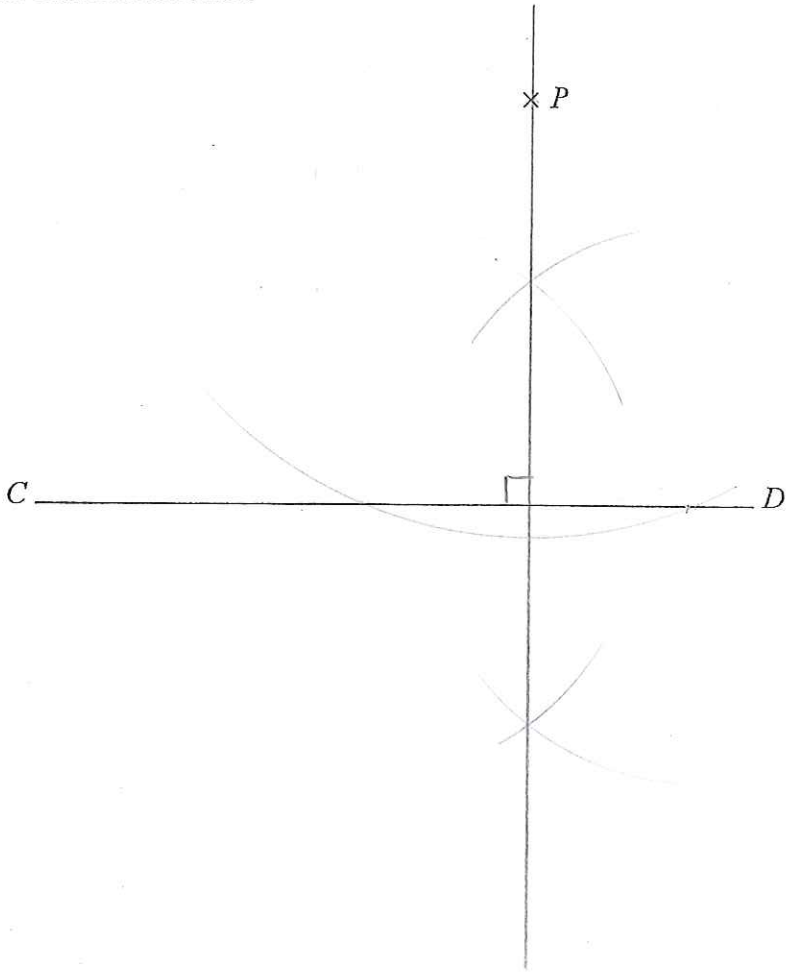
$= 2\frac{4}{12}$

$= 2\frac{1}{3}$

$2\frac{1}{3}$

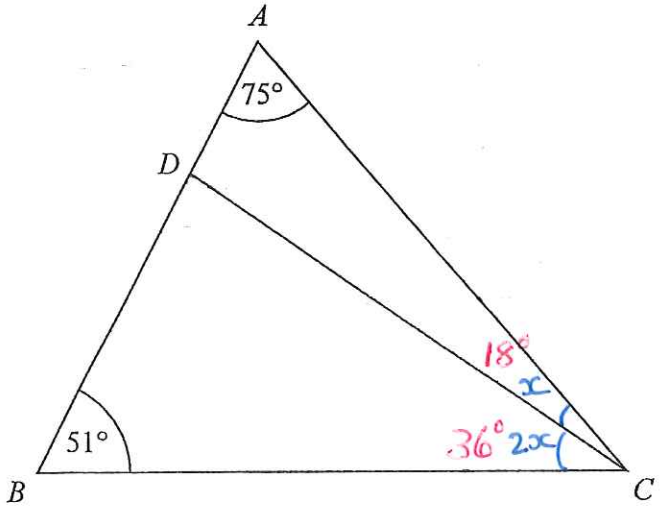
(Total for Question 3 is 3 marks)

4 Use a ruler and compasses to construct the line from the point P perpendicular to the line CD.
You must show **all** construction lines.



(Total for Question 4 is 2 marks)

5 The diagram shows triangle *ABC*.



ADB is a straight line.

the size of angle *DCB* : the size of angle *ACD* = 2 : 1

Work out the size of angle *BDC*.

Let angle ACD = x

∴ angle DCB = $2x$

and angle ACB = $3x$

consider the triangle ABC

angles in a triangle add up to 180°

$75 + 51 + 3x = 180$

$126 + 3x = 180$ [−126

$3x = 54$ [÷3

$x = 18$

75
+ 51
126

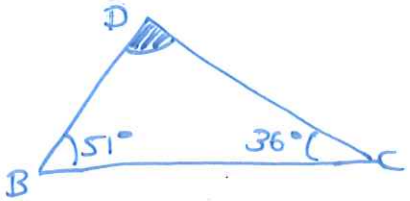
7 1
1 8 0
− 1 2 6
5 4

1 8
3) 5 4

So angle ACD = 18°

and angle DCB = 36°

now consider the triangle BCD



51° + 36° = 87°

angle BDC

= 180° − 87°

= 93°

° 1 7 1
1 8 0
− 8 7
9 3

93 °

(Total for Question 5 is 4 marks)

6 4 red bricks have a mean weight of 5 kg.
5 blue bricks have a mean weight of 9 kg.
1 green brick has a weight of 6 kg.

Donna says,

"The mean weight of the 10 bricks is less than 7 kg."

Is Donna correct?
You must show how you get your answer.

$\frac{\text{total weight of 4 Red bricks}}{4} = 5\text{kg}$ ∴ total weight of 4 Red Bricks = 20 kg

$\frac{\text{total weight of 5 Blue bricks}}{5} = 9\text{kg}$ ∴ total weight of 5 Blue Bricks = 45 kg

mean weight of 4 Red Bricks, 5 Blue Bricks and 1 green brick

$= \frac{20\text{ kg} + 45\text{ kg} + 6\text{ kg}}{10}$

$= \frac{71\text{ kg}}{10}$

$= 7.1\text{ kg}$

Donna is not correct. The mean weight of the 10 bricks is 7.1 kg which is not less than 7 kg

(Total for Question 6 is 3 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

7 (a) Simplify $(p^2)^5$

$= p^{2\times5}$

p^{10}

(1)

(b) Simplify $12x^7y^3 \div 6x^3y$

$$\frac{2\ 12xxxxxxxyyy}{6xxxy\ 1}$$

or

$$\frac{12}{6}\ \frac{x^7}{x^3}\ \frac{y^3}{y^1}$$

$= 2x^{7-3}y^{3-1}$

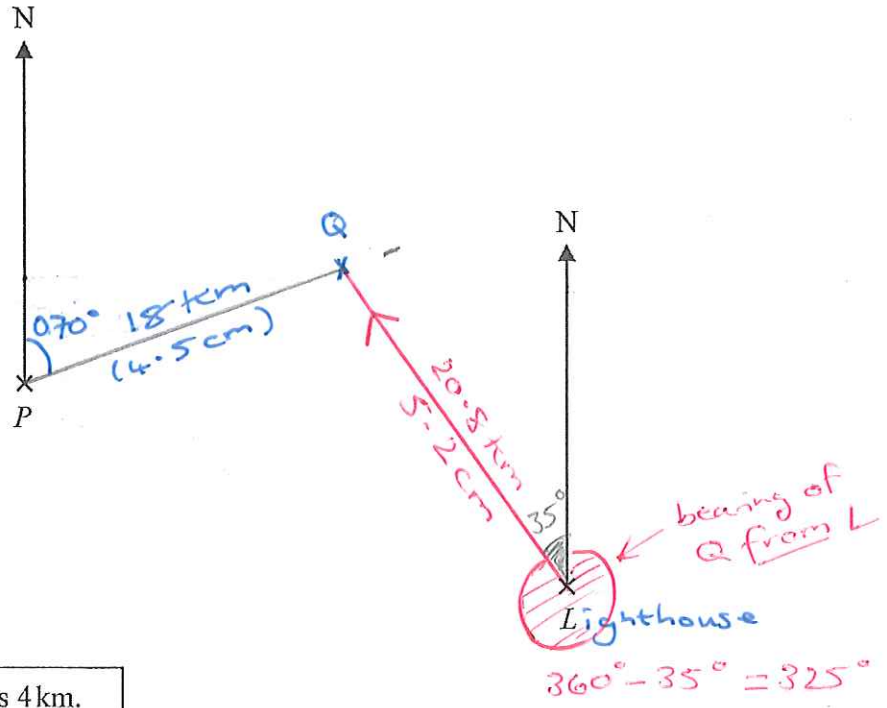
$= 2x^4y^2$

$2x^4y^2$

(2)

(Total for Question 7 is 3 marks)

8 The accurate scale drawing shows the positions of port P and a lighthouse L.



Scale: 1 cm represents 4 km.

Aleena sails her boat from port P on a bearing of 070°

She sails for $1\frac{1}{2}$ hours at an average speed of 12 km/h to a port Q.

Find

(i) the distance, in km, of port Q from lighthouse L,

(ii) the bearing of port Q from lighthouse L.

Port P → Port Q

S = 12 km/h

D

T = 1.5 hours

D = S × T
= 12 × 1.5
= 18 km

52
× 4
208

Scale 1 cm = 4 km

$\frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$ cm

Lighthouse L → Port Q

1 cm : 4 km

× 5.2 ↓ ↓ × 5.2

5.2 cm : 20.8 km

answers in range 20 to 23 accepted

distance QL = 20.8 km

bearing of Q from L = 325 °

answers in range 317 to 330 accepted

(Total for Question 8 is 5 marks)

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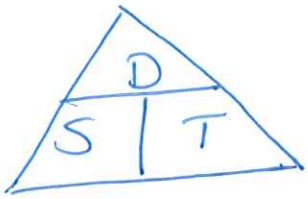
DO NOT WRITE IN THIS AREA

9 A car travels for 18 minutes at an average speed of 72 km/h.



(a) How far will the car travel in these 18 minutes?

$S = 72$ km / hour

$D =$

$T = 18$ minutes

change minutes into hours

18 minutes $= \frac{18}{60}$ of an hour

$= \frac{3}{10}$ of an hour

$D = S \times T$

$= \frac{72}{1} \times \frac{3}{10}$

$= \frac{216}{10}$

72 × 3 = 216

21·6 km

(2)

David says,

"72 kilometres per hour is faster than 20 metres per second."

(b) Is David correct?
You must show how you get your answer.

72 km = 72000 m (×1000)

1 hour = 3600 seconds (×60×60)

72 km : 1 hour

72 000 m : 3600 seconds (÷100, ÷100)

720 m : 36 seconds (÷36, ÷36)

20 m : 1 second

72 km/h is the same as 20 m/s

David is not correct

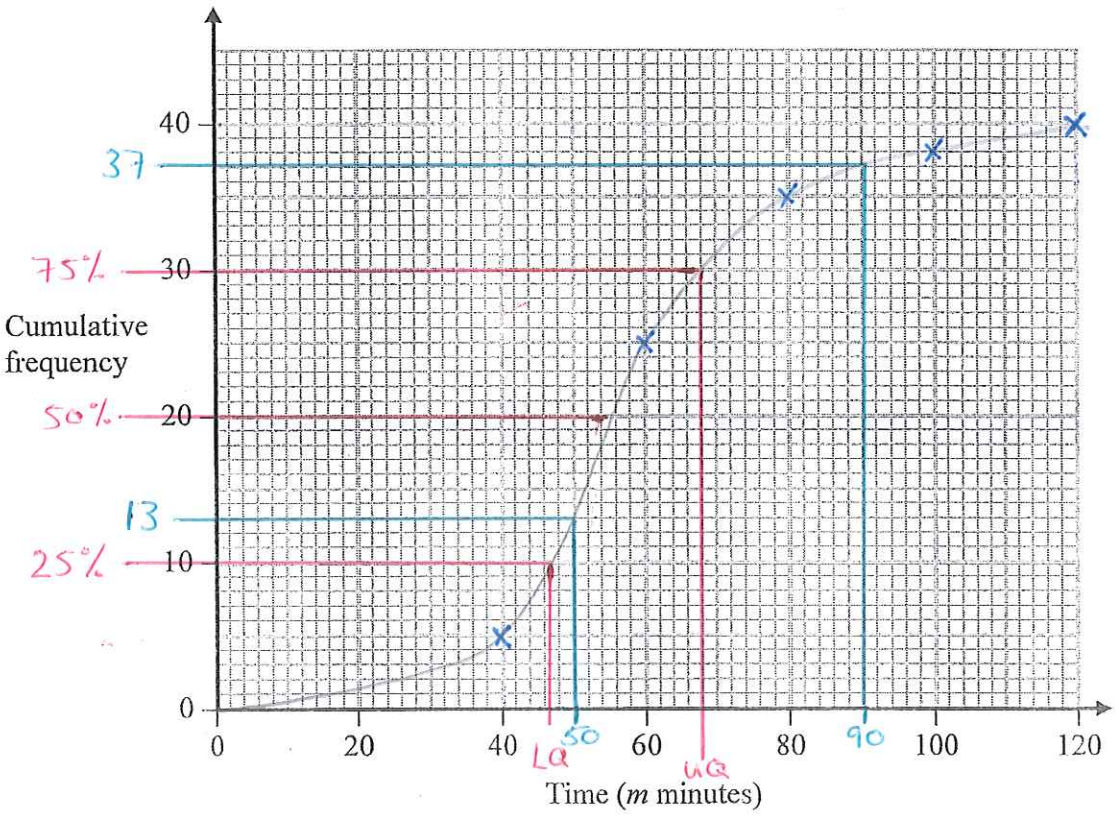
(2)

(Total for Question 9 is 4 marks)

10 The cumulative frequency table shows information about the times, in minutes, taken by 40 people to complete a puzzle.

Time (m minutes)	Cumulative frequency
$20 < m \leqslant 40$	5
$20 < m \leqslant 60$	25
$20 < m \leqslant 80$	35
$20 < m \leqslant 100$	38
$20 < m \leqslant 120$	40

(a) On the grid below, draw a cumulative frequency graph for this information.



(2)

(b) Use your graph to find an estimate for the interquartile range.

Lower quartile ≈ 47 minutes
upper quartile ≈ 68 minutes

IQR = UQ − LQ
= 68 − 47
= 21 minutes

answers in range 21 to 28 accepted

21 minutes

(2)

One of the 40 people is chosen at random.

(c) Use your graph to find an estimate for the probability that this person took between 50 minutes and 90 minutes to complete the puzzle.

37 − 13 = 24 people took between 50 and 90 minutes

37
−13
24

P(a person takes between 50 and 90 minutes)

$= \frac{24}{40}$ or $\frac{3}{5}$

answers in range $\frac{19}{40}$ to $\frac{24}{40}$ accepted

(2)

(Total for Question 10 is 6 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

11 There are p counters in a bag.
12 of the counters are yellow.

Shafiq takes at random 30 counters from the bag.
5 of these 30 counters are yellow.

Work out an estimate for the value of p.

"Capture – Recapture"

$\frac{12}{p} = \frac{5}{30}$ (×2·4)

$\frac{12}{5} = 2\frac{2}{5} = 2{\cdot}4$

$P \approx 30 \times 2{\cdot}4$

$= 72$ counters

$30 \times 2 = 60$

$30 \times 0{\cdot}4$

$= 30 \times \frac{4}{10}$

$= \frac{30}{1} \times \frac{2}{5}$

$= \frac{60}{5} = 12$

(Total for Question 11 is 2 marks)

12 $T = \frac{q}{2} + 5$

Here is Spencer's method to make q the subject of the formula.

$2 \times T = q + 5$

$q = 2T - 5$

What mistake did Spencer make in the first line of his method?

He should have subtracted 5 from both sides before multiplying by 2

(Total for Question 12 is 1 mark)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

13 (a) Write $\frac{5}{x+1} + \frac{2}{3x}$ as a single fraction in its simplest form.

need to find a common denominator
use $3x(x+1)$

$$\frac{5(3x)}{(x+1)(3x)} + \frac{2(x+1)}{3x(x+1)}$$

$$= \frac{5(3x) + 2(x+1)}{(x+1)(3x)}$$

$$= \frac{15x + 2x + 2}{(x+1)(3x)} = \frac{17x + 2}{3x(x+1)}$$

(2)

(b) Factorise $(x + y)^2 + 3(x + y)$

Let $a = (x+y)$

$$a^2 + 3a$$
$$= a(a+3)$$

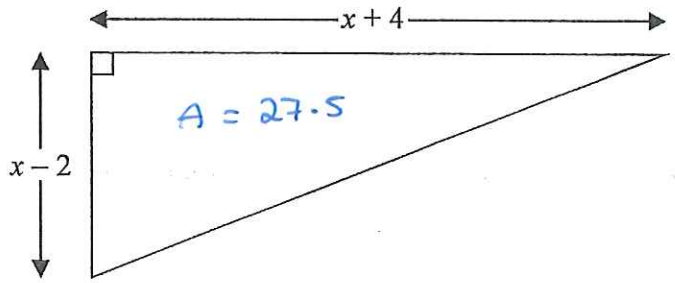
so we have

$$(x+y)(x+y+3)$$

(1)

(Total for Question 13 is 3 marks)

14 The diagram shows a right-angled triangle.



All the measurements are in centimetres.

The area of the triangle is 27.5 cm²

Work out the length of the shortest side of the triangle.
You must show all your working.

Area triangle $= \frac{(x-2)(x+4)}{2}$

$= \frac{x^2 + 4x - 2x - 8}{2}$

$= \frac{x^2 + 2x - 8}{2}$

$\therefore \frac{x^2}{2} + x - 4 = 27.5$ [$\times 2$

$x^2 + 2x - 8 = 55$ [-55

$x^2 + 2x - 63 = 0$

$(x - 7)(x + 9) = 0$

either $x - 7 = 0$ or $x + 9 = 0$

$x = 7$ $x = -9$

a length cannot be negative $\therefore x = 7$

shortest side $x - 2$

$= 7 - 2$

$= 5$

27.5
27.5
55.0
1 1

5 ..... cm

(Total for Question 14 is 4 marks)

15 Express $0.4\dot{1}\dot{8}$ as a fraction.
You must show all your working.

let $x = 0.418181818\ldots$

2 digits recur $\times 100$

$100x = 41.818181\,81\ldots$

$-\quad x = 0.418181\,81\ldots$

$99x = 41.4$

$x = \frac{41.4}{99} \times 10$

$x = \frac{414}{990}$

$\frac{414}{990}$

(Total for Question 15 is 3 marks)

16 (a) Rationalise the denominator of $\frac{22}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}}$

Give your answer in its simplest form.

$= \frac{22\sqrt{11}}{11}$

$2\sqrt{11}$

(2)

(b) Show that $\frac{\sqrt{3}}{2\sqrt{3}-1}$ can be written in the form $\frac{a+\sqrt{3}}{b}$ where a and b are integers.

$\frac{\sqrt{3}(2\sqrt{3}+1)}{(2\sqrt{3}-1)(2\sqrt{3}+1)}$

Numerator

$\sqrt{3}(2\sqrt{3}+1)$

$= 6 + \sqrt{3}$

Denominator

$(2\sqrt{3}-1)(2\sqrt{3}+1)$

$= 12 + 2\sqrt{3} - 2\sqrt{3} - 1$

$= 11$

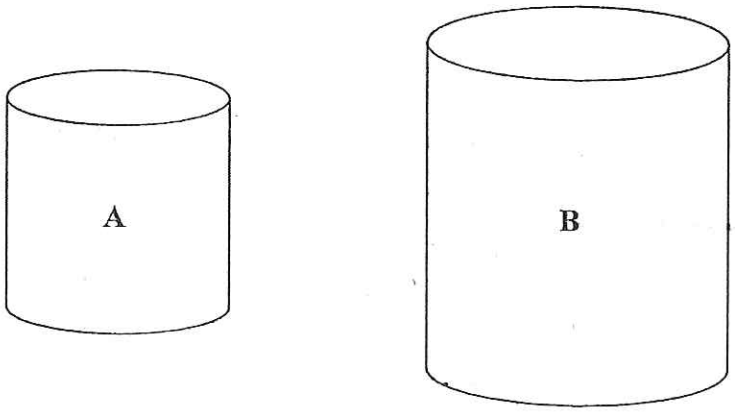
$= \frac{6+\sqrt{3}}{11}$

$a = 6,\ b = 11$

(3)

(Total for Question 16 is 5 marks)

17 **A** and **B** are two similar cylindrical containers.



the surface area of container **A** : the surface area of container **B** = 4 : 9

Tyler fills container **A** with water.
She then pours all the water into container **B**.
Tyler repeats this and stops when container **B** is full of water.

Work out the number of times that Tyler fills container **A** with water.
You must show all your working.

Area Scale Factor $k^2 = \frac{9}{4}$

length Scale Factor $k = \frac{3}{2}$

volume Scale Factor $k^3 = \frac{27}{8} = 3\frac{3}{8}$

∴ Tyler will fill container A with water 4 times

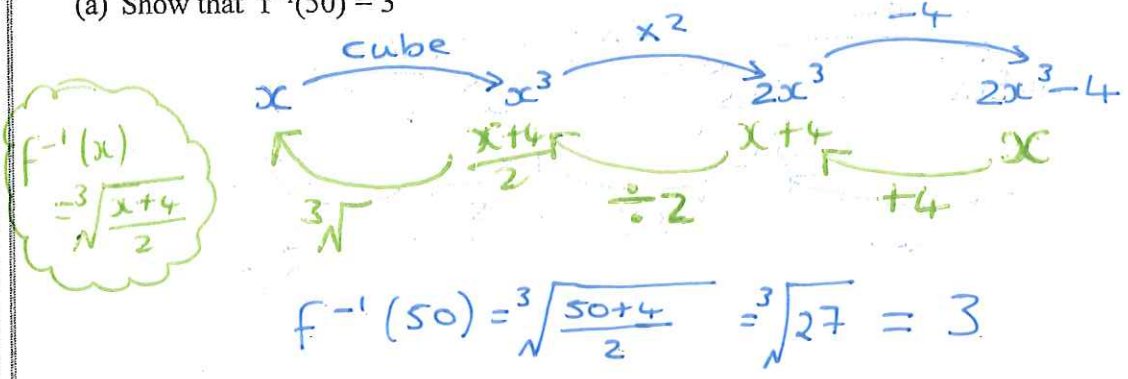
4

(Total for Question 17 is 4 marks)

18 The function f is given by

$$f(x) = 2x^3 - 4$$

(a) Show that $f^{-1}(50) = 3$



$$f^{-1}(50) = \sqrt[3]{\frac{50+4}{2}} = \sqrt[3]{27} = 3$$

(2)

The functions g and h are given by

$$g(x) = x + 2 \quad \text{and} \quad h(x) = x^2$$

(b) Find the values of x for which

$$hg(x) = 3x^2 + x - 1$$

$$h[g(x)] = h(x+2)$$
$$= (x+2)^2$$
$$= x^2 + 4x + 4$$

$$hg(x) = 3x^2 + x - 1$$

$$\therefore \quad x^2 + 4x + 4 = 3x^2 + x - 1 \qquad [-x^2$$
$$4x + 4 = 2x^2 + x - 1 \qquad [-4x$$
$$4 = 2x^2 - 3x - 1 \qquad [-4$$
$$0 = 2x^2 - 3x - 5$$

$$2x^2 - 3x - 5 = 0$$
$$(2x - 5)(x + 1) = 0$$

$2x - 5 = 0$ or $x + 1 = 0$

$2x = 5$ $\quad x = -1$

$x = \frac{5}{2}$

$x = \frac{5}{2}, -1$

(4)

(Total for Question 18 is 6 marks)

19 Given that $9^{-\frac{1}{2}} = 27^{\frac{1}{4}} \div 3^{x+1}$
find the exact value of x.

$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3} = 3^{-1}$ write each term as a power of 3

$27^{\frac{1}{4}} = (3^3)^{\frac{1}{4}} = 3^{\frac{3}{4}}$

$3^{-1} = 3^{\frac{3}{4}} \div 3^{x+1}$

$3^{-1} = 3^{\frac{3}{4} - (x+1)}$

comparing powers of 3 gives

$-1 = \frac{3}{4} - x - 1$

$-1 = -\frac{1}{4} - x$ $[+\frac{1}{4}$

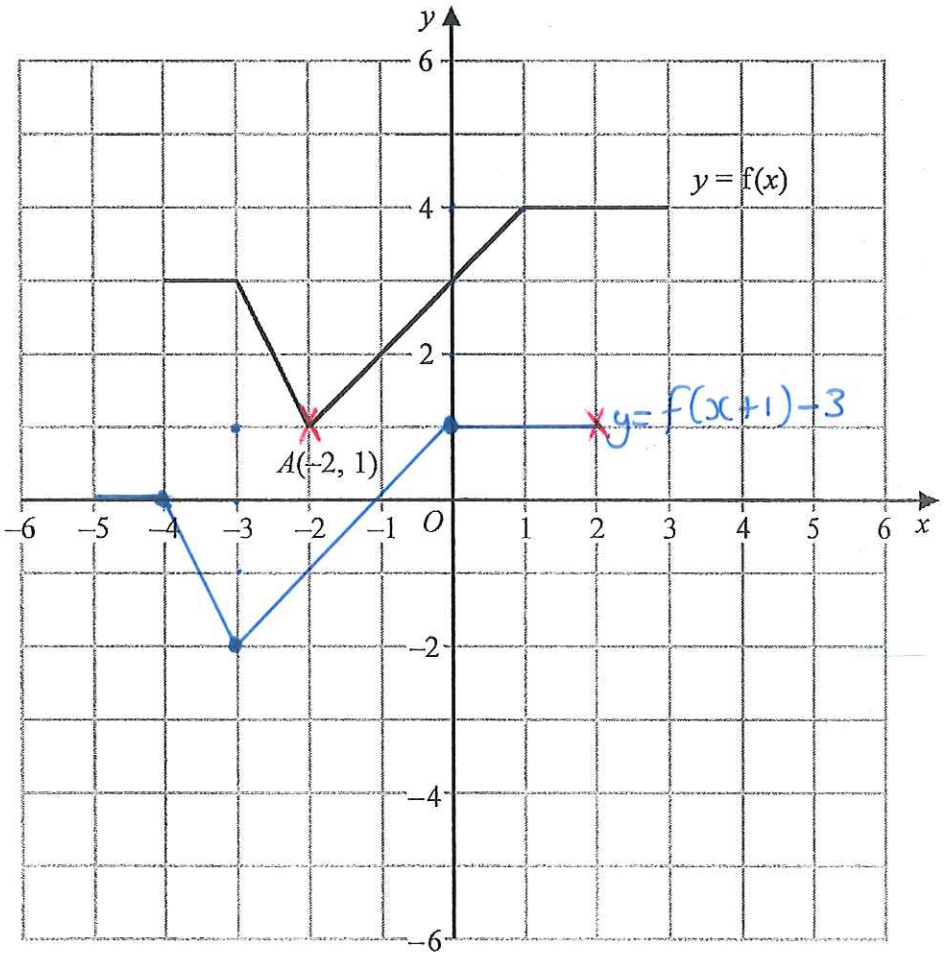
$-\frac{3}{4} = -x$ $[\times -1$

$\frac{3}{4} = x$

$x = \frac{3}{4}$

(Total for Question 19 is 3 marks)

20 The graph of $y = f(x)$ is shown on the grid.



(a) On the grid, draw the graph with equation $y = f(x + 1) - 3$ ← down 3

left 1

(2)

Point $A(-2, 1)$ lies on the graph of $y = f(x)$.

When the graph of $y = f(x)$ is transformed to the graph with equation $y = f(-x)$, point A is mapped to point B.

(b) Write down the coordinates of point B.

reflection in y-axis

(2 , 1)

(1)

(Total for Question 20 is 3 marks)

21 Sketch the graph of

$$y = 2x^2 - 8x - 5$$

y-intercept

showing the coordinates of the turning point and the exact coordinates of any intercepts with the coordinate axes.

$y = 2(x^2 - 4x) - 5$

$= 2[(x-2)^2 - 4] - 5$

$= 2(x-2)^2 - 8 - 5$

$= 2(x-2)^2 - 13$

right 2 down 13

vertical stretch SF 2

x-intercepts

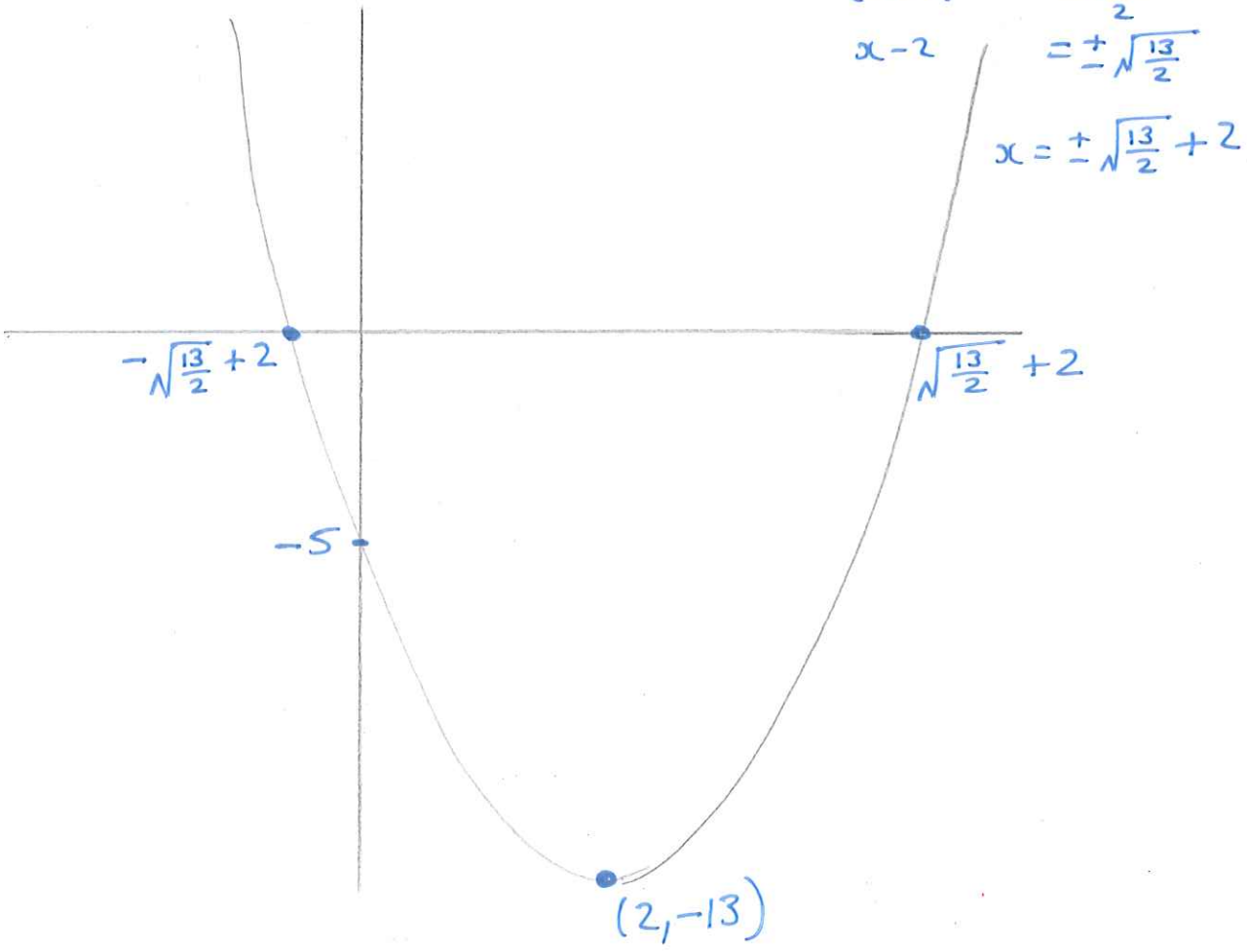
$2(x-2)^2 - 13 = 0$

$2(x-2)^2 = 13$

$(x-2)^2 = \frac{13}{2}$

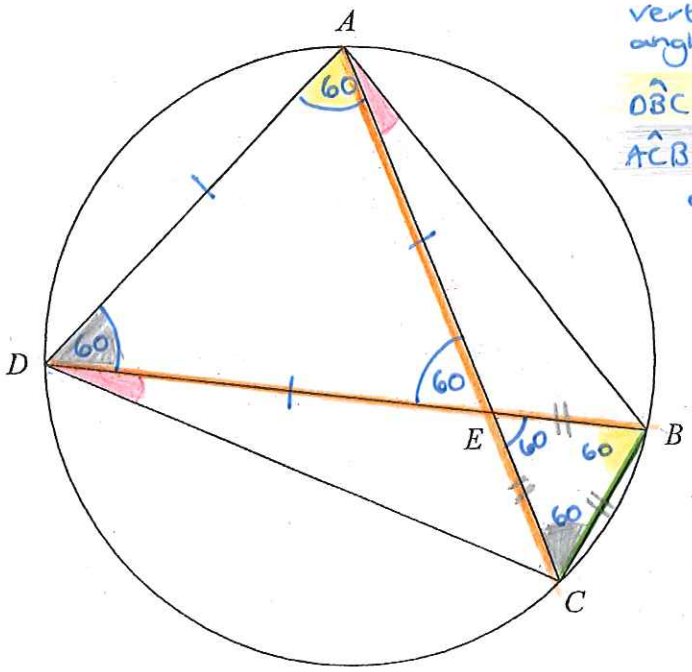
$x - 2 = \pm\sqrt{\frac{13}{2}}$

$x = \pm\sqrt{\frac{13}{2}} + 2$



(Total for Question 21 is 5 marks)

22 *A*, *B*, *C* and *D* are four points on a circle.



$C\hat{E}B = A\hat{E}D = 60°$ vertically opposite angles are equal

$D\hat{B}C = C\hat{A}D = 60°$

$A\hat{C}B = A\hat{D}B = 60°$ angles in the same segment are equal

triangle BCE is an equilateral triangle

AEC and *DEB* are straight lines.

Triangle *AED* is an equilateral triangle.

Prove that triangle *ABC* is congruent to triangle *DCB*.

(A) angle COB = angle CAB angles in the same segment are equal

(S) BC is a shared side

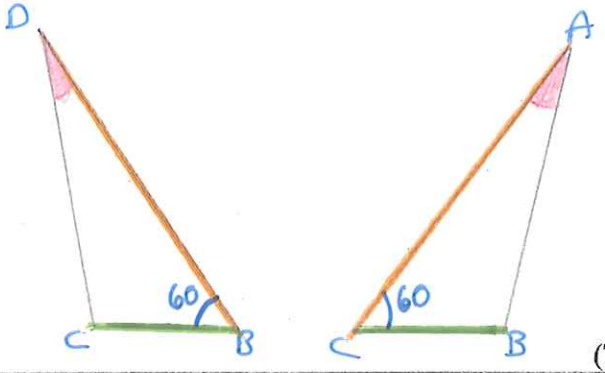
AE = DE (sides of an equilateral triangle)

and

EC = EB (sides of an equilateral triangle)

(S) ∴ AC = DB

(A) $D\hat{B}C = A\hat{C}B = 60°$



∴ triangle ABC is congruent to triangle DCB SAS or ASA

(Total for Question 22 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS