

LP GS GCSE Mock 2 Paper 3

HIGHER TIER

Wednesday 4th March 2020 AFTERNOON

Name _____

LTH

Maths Teacher _____

Worked Solutions

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may be used.**
- If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.



Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

TOTAL MARK out 80

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 (a) Expand and simplify $(x+5)(x-9)$

$$= x^2 - 9x + 5x - 45$$

$$= x^2 - 4x - 45$$

- (b) Factorise fully $9x^2 + 6x$

$$= 3x(3x + 2)$$

$$= 3 \times 3 \times x + 3 \times 2 \times x$$

(2)

(2)

(Total for Question 1 is 4 marks)

- 2 (a) Use your calculator to work out $\frac{29^2 - 4.6}{\sqrt{35 - 1.9^3}}$

Write down all the figures on your calculator display.

(2)
157.668255

- (b) Write your answer to part (a) correct to 4 significant figures.

(1)
157.7

(Total for Question 2 is 3 marks)

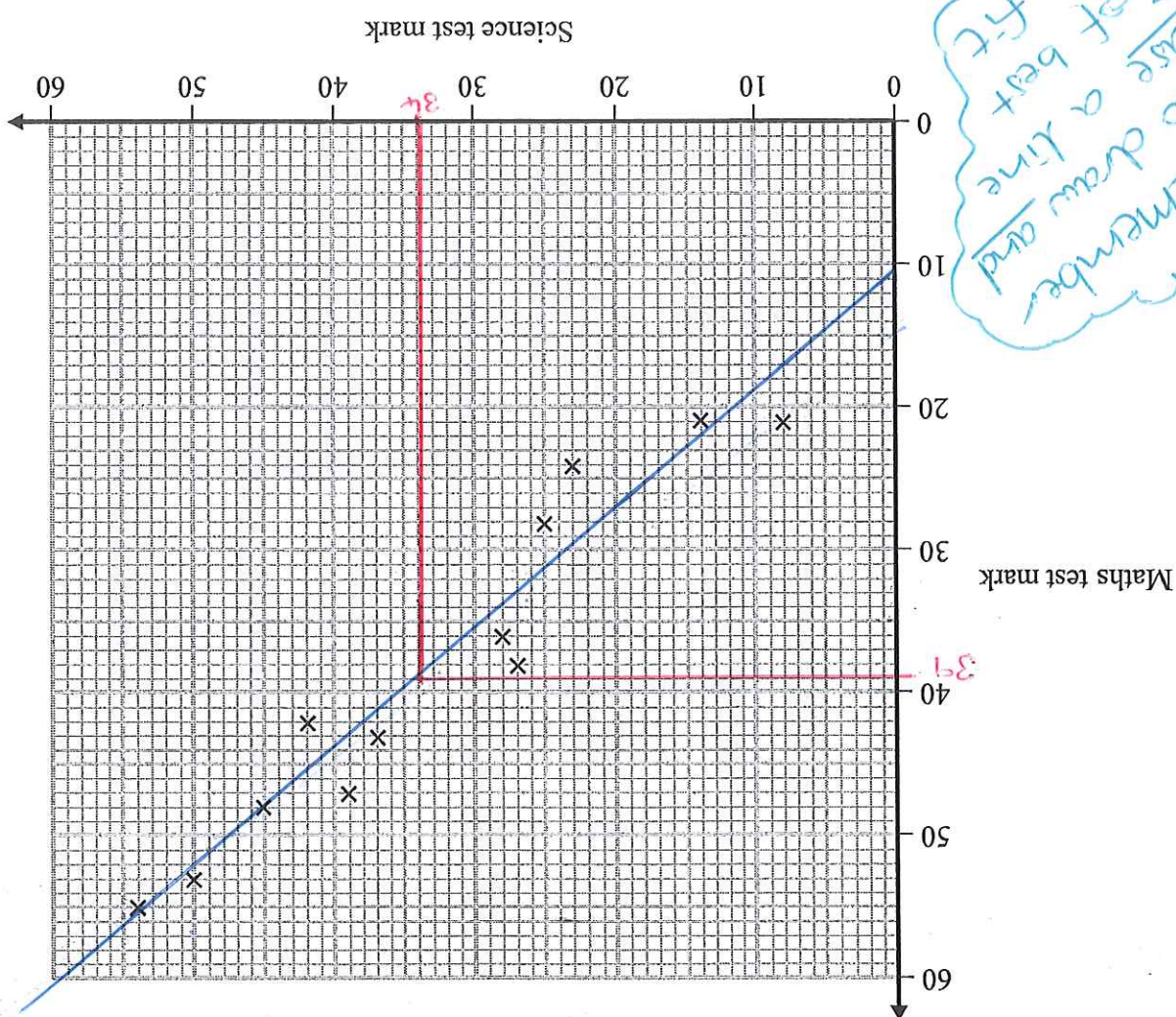


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3 The scatter graph shows information about the marks a group of students got in a Science test and in a Maths test.



remember to draw and use a line of best fit

Jamie got a mark of 34 in the Science test.

Using the scatter graph, find an estimate for Jamie's mark in the Maths test.

(Total for Question 3 is 2 marks)

39



P 5 8 8 7 6 R A 0 3 2 4



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4 The table gives information about the times taken, in seconds, by 18 students to run a race.

Time (t seconds)	Frequency
$5 < t \leq 10$	1
$10 < t \leq 15$	2
$15 < t \leq 20$	7
$20 < t \leq 25$	8

mid point

Work out an estimate for the mean time.
Give your answer correct to 3 significant figures.

$$\text{mean} \approx \frac{7.5 \times 1 + 12.5 \times 2 + 17.5 \times 7 + 22.5 \times 8}{18}$$

$$= \frac{335}{18}$$

$$= 18.611111$$

$$= 18.6 \text{ (3sf)}$$

..... seconds

(Total for Question 4 is 3 marks)

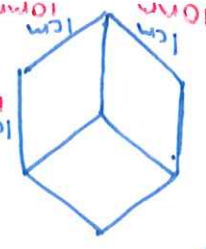
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5 Write 37 cm^3 in mm^3

Remember - ...



Volume = $1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$
 $= 1 \text{ cm}^3$
 Volume = $10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$
 $= 1000 \text{ mm}^3$

(Total for Question 5 is 1 mark)

$1 \text{ cm}^3 = 1000 \text{ mm}^3$

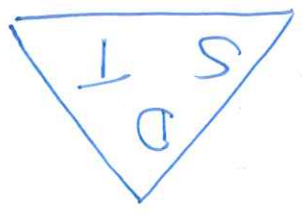
$\xrightarrow{\times 1000} \text{mm}^3$

$37 \text{ cm}^3 = 37000 \text{ mm}^3$

37000 mm^3

6 Nimer was driving to a hotel. He looked at his Sat Nav at 13 30

Time	13 30
Distance to destination	65 miles



Nimer arrived at the hotel at 14 48

Work out the average speed of the car from 13 30 to 14 48
 You must show all your working.

$S =$
 $D = 65 \text{ miles}$
 $T = 1 \text{ hour } 18 \text{ mins}$
 $= 1 \frac{3}{5} \text{ hours}$
 $= 1.6 \text{ hours}$

Average Speed = $\frac{D}{T}$
 $= \frac{65}{1.6}$

if you prefer you can enter 1018 on your calculator using the time button [09 99]

50 mph

(Total for Question 6 is 4 marks)





(Total for Question 7 is 3 marks)

(1)

A 6.21200000
B 4.73000000
6.21,200,000
4.730,000,000

B is bigger

No. 4.73 will be multiplied by 10 nine times whereas
6.212 will only be multiplied by 10 eight times making
it smaller than B

(c) Is Asma correct?
You must give a reason for your answer.

"6.212 is bigger than 4.73 so A is bigger than B."

She says,

$$A = 6.212 \times 10^8 \text{ and } B = 4.73 \times 10^9$$

Asma was asked to compare the following two numbers.

(1)

$$0.00496$$

$$0.00496$$

(b) Write 4.96×10^{-3} as an ordinary number.

(1)

$$3.246 \times 10^7$$

$$3.246 \times 10^7$$

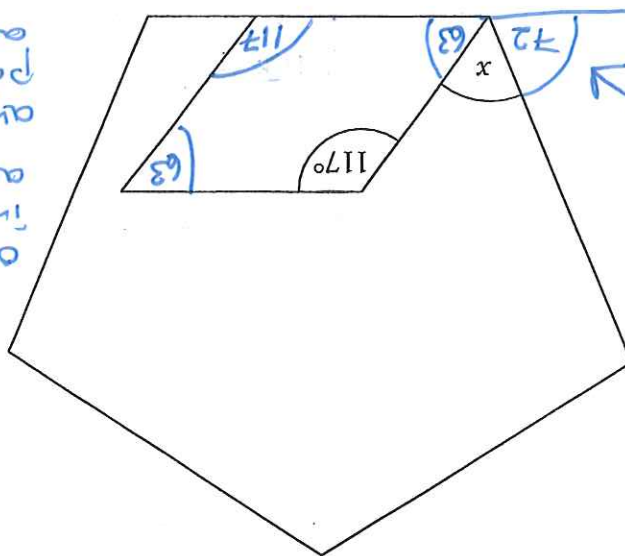
7 (a) Write 32 460 000 in standard form.

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8 The diagram shows a regular pentagon and a parallelogram.



Exterior angle
of pentagon
 $= 360^\circ \div 5$

Work out the size of the angle marked x.
You must show all your working.

$$x = 180 - 72 - 63 = 45^\circ$$

angles on a straight
line add up to 180°

opposite angles
in a parallelogram
are equal
angles in a
parallelogram
add up to 360°

(Total for Question 8 is 4 marks)

45



P 5 8 8 7 6 R A 0 7 2 4

Turn over



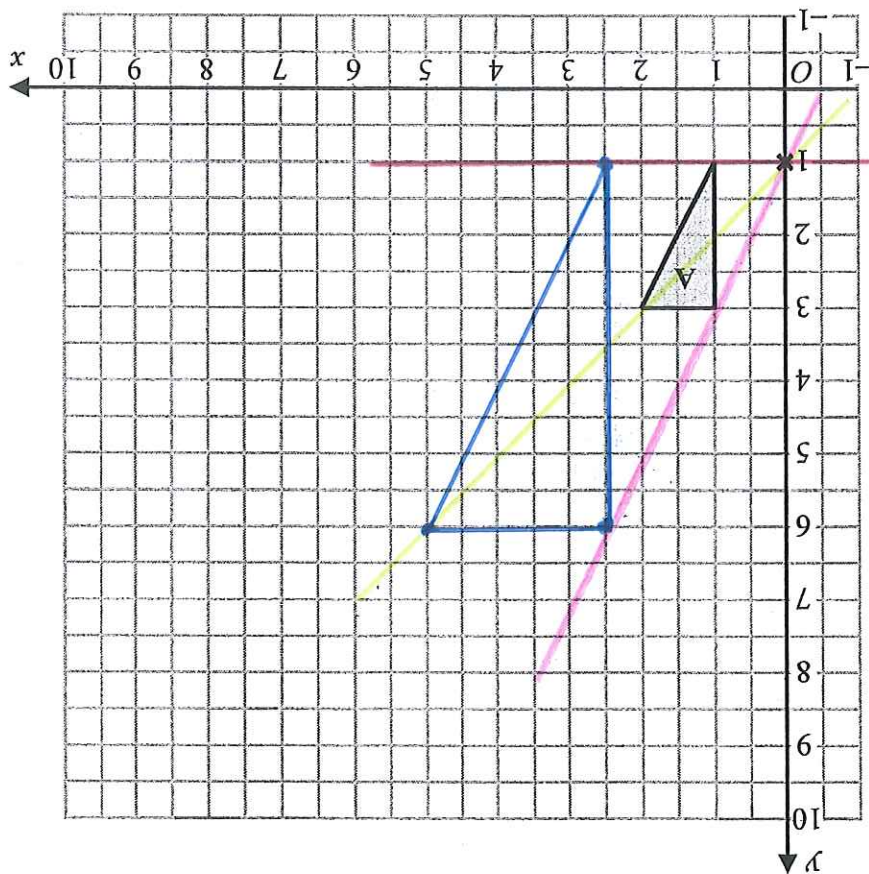
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(Total for Question 9 is 2 marks)

Enlarge triangle A by scale factor 2.5 with centre (0, 1)



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10 (a) Solve $\frac{9}{x+7} = 11 - x$

[x 7

$$9 + x = 7(11 - x)$$

$$9 + x = 77 - 7x$$

$$9 + 8x = 77$$

$$8x = 68$$

$$x = \frac{68}{8}$$

$$\text{or } x = 8.5$$

$$[+7x$$

$$[-9$$

$$[\div 8$$

(b) Simplify $\frac{4(y+3)^2}{(y+3)^2} = \frac{4(y+3)(y+3)}{(y+3)(y+3)}$

$$= 4(y+3)$$

$$= 4y + 12$$

(Total for Question 10 is 4 marks)

(1)

$$4y + 12$$

(3)

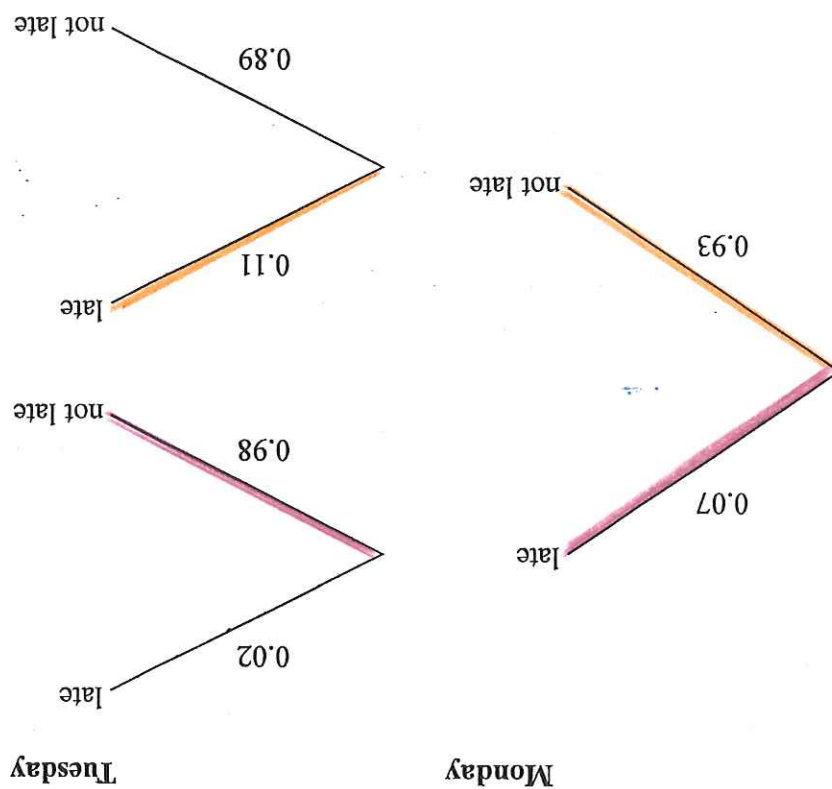
$$x = 8.5$$



P 5 8 8 7 6 R A 0 9 2 4

Turn over

11 The probability tree diagram shows the probabilities that Bismah will be late for work on two days next week.



Calculate the probability that Bismah will be late on exactly one of the two days.

$$\begin{aligned}
 &= P(\text{Late Monday and not Late Tuesday}) \\
 &\quad \text{OR not Late Monday and Late Tuesday} \\
 &= (0.07 \times 0.98) + (0.93 \times 0.11) \\
 &= 0.1709
 \end{aligned}$$

X along branches
+ between branches

(Total for Question 11 is 3 marks)



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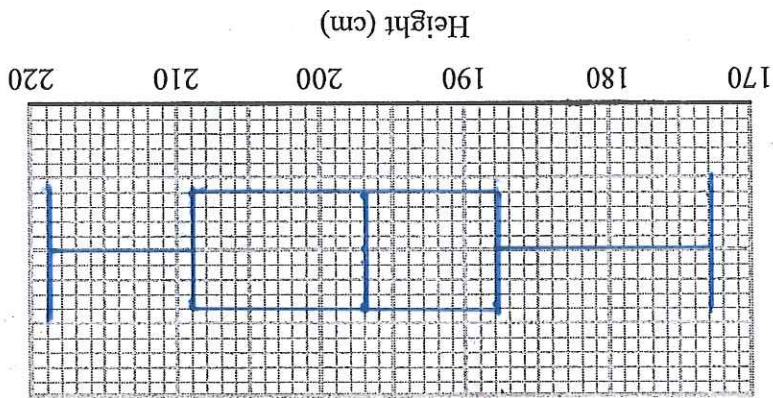
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12 The stem and leaf diagram shows information about the heights, in cm, of 23 sunflowers.

17	3	4	9			
18	6	8	8	7		
19	0	0	1	4	6	7
20	1	4	7	7	9	9
21	4	8	8			9

On the grid, draw a box plot for this information.

Box Plot to show the height of sunflowers



(Total for Question 12 is 3 marks)

Key: 17|3 represents 173 cm

Median = 197

Lower Quartile = 188

Upper Quartile = 209

Smallest = 173

Largest = 219

$$\frac{n+1}{2} = 12$$

$$n = 23$$

Median is 12th value



P 5 8 8 7 6 R A 0 1 1 2 4

13 Liquid A and liquid B are mixed together in the ratio 2:13 by volume to make liquid C.

Liquid A has density 1.21 g/cm^3
Liquid B has density 1.02 g/cm^3

A cylindrical container is filled completely with liquid C.
The cylinder has radius 3 cm and height 25 cm.

Work out the mass of the liquid in the container.
Give your answer correct to 3 significant figures.
You must show all your working.

$$\text{Liquid A} + \text{Liquid B} = \text{Liquid C}$$

$$D = 1.21 \text{ g/cm}^3 \quad D = 1.02 \text{ g/cm}^3$$

$$M = \frac{363}{10} \pi \text{ grams} + M = \frac{1989}{10} \pi \text{ grams} = M = 738.9025 \text{ grams}$$

$$V = 30 \pi \text{ cm}^3 + V = 195 \pi \text{ cm}^3 = V = 225 \pi \text{ cm}^3$$

Step 2

The volume of liquid C is $\frac{5}{2}$ liquid A and $\frac{13}{15}$ liquid B

$$\frac{15}{225\pi} = 15\pi \quad \frac{15}{225\pi} = 15\pi$$

$$\text{Liquid A} = 2 \times 15\pi = 30\pi \text{ cm}^3$$

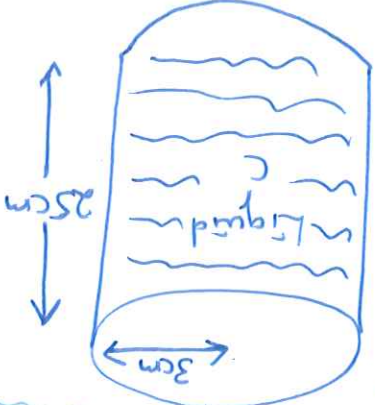
$$\text{Liquid B} = 13 \times 15\pi = 195\pi \text{ cm}^3$$

Step 3

$$\text{Liquid A mass} = D \times V = \frac{363}{10} \pi \text{ grams}$$

$$\text{Liquid B mass} = D \times V = \frac{1989}{10} \pi \text{ grams}$$

$$\text{Liquid C mass} = \text{Liquid A mass} + \text{Liquid B mass} = 738.9025 \pi \text{ grams}$$



Step 1

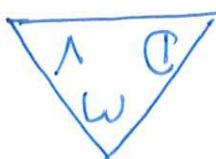
$$\text{Volume cylinder} = \text{area cross section} \times \text{height}$$

$$= \pi r^2 \times h$$

$$= \pi \times 3^2 \times 25$$

$$= 225 \pi$$

TIP
Work in terms of π to avoid decimals



739 (3sf)

(Total for Question 13 is 4 marks)



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14 A group of people went to a restaurant.
Each person chose one starter and one main course.

starter	main course
soup	lasagne
prawns	curry

the number of people who chose soup : the number of people who chose prawns = 2 : 3

Of those who chose soup,

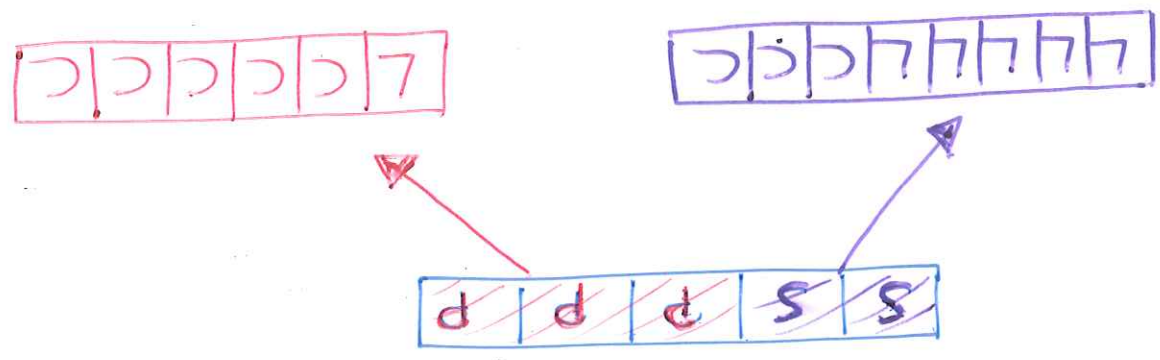
the number of people who chose lasagne : the number of people who chose curry = 5 : 3

Of those who chose prawns,

the number of people who chose lasagne : the number of people who chose curry = 1 : 5

What fraction of the people chose curry?

You must show how you get your answer.



$\frac{3}{2}$ of $\frac{8}{5}$ Curry

$\frac{5}{3}$ of $\frac{6}{5}$

$$= \frac{3}{2} \times \frac{8}{5} + \frac{6}{5} \times \frac{3}{5}$$

$$= \frac{40}{6} + \frac{15}{30}$$

$$= \frac{13}{20} \text{ of the people chose curry}$$

$$\frac{13}{20}$$

(Total for Question 14 is 4 marks)





(Total for Question 16 is 3 marks)

16 y is inversely proportional to the square of x .
 $y = 8$ when $x = 2.5$
 Find the negative value of x when $y = \frac{8}{9}$

$$y \propto \frac{1}{x^2}$$

$$y = \frac{k}{x^2}$$

$$8 = \frac{k}{2.5^2}$$

$$50 = k$$

$$y = \frac{50}{x^2}$$

negative value

$$x = -7.5$$

$$y = \frac{50}{x^2}$$

when $y = \frac{8}{9}$

$$\frac{8}{9} = \frac{50}{x^2}$$

$$8x^2 = 50 \times 9$$

$$8x^2 = 450$$

$$x^2 = \frac{450}{8}$$

$$x = \pm \sqrt{\frac{450}{8}}$$

$$x = \pm \frac{15}{2}$$

(Total for Question 15 is 3 marks)

15 Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4

Let $2n$ and $2n+2$ be consecutive even numbers

$$(2n+2)^2$$

$$= (2n+2)(2n+2)$$

$$= 4n^2 + 4n + 4n + 4$$

$$= 4n^2 + 8n + 4$$

$$(2n)^2 + (2n+2)^2$$

$$= 4n^2 + 4n^2 + 8n + 4$$

$$= 8n^2 + 8n + 4$$

$$= 4(2n^2 + 2n + 1)$$

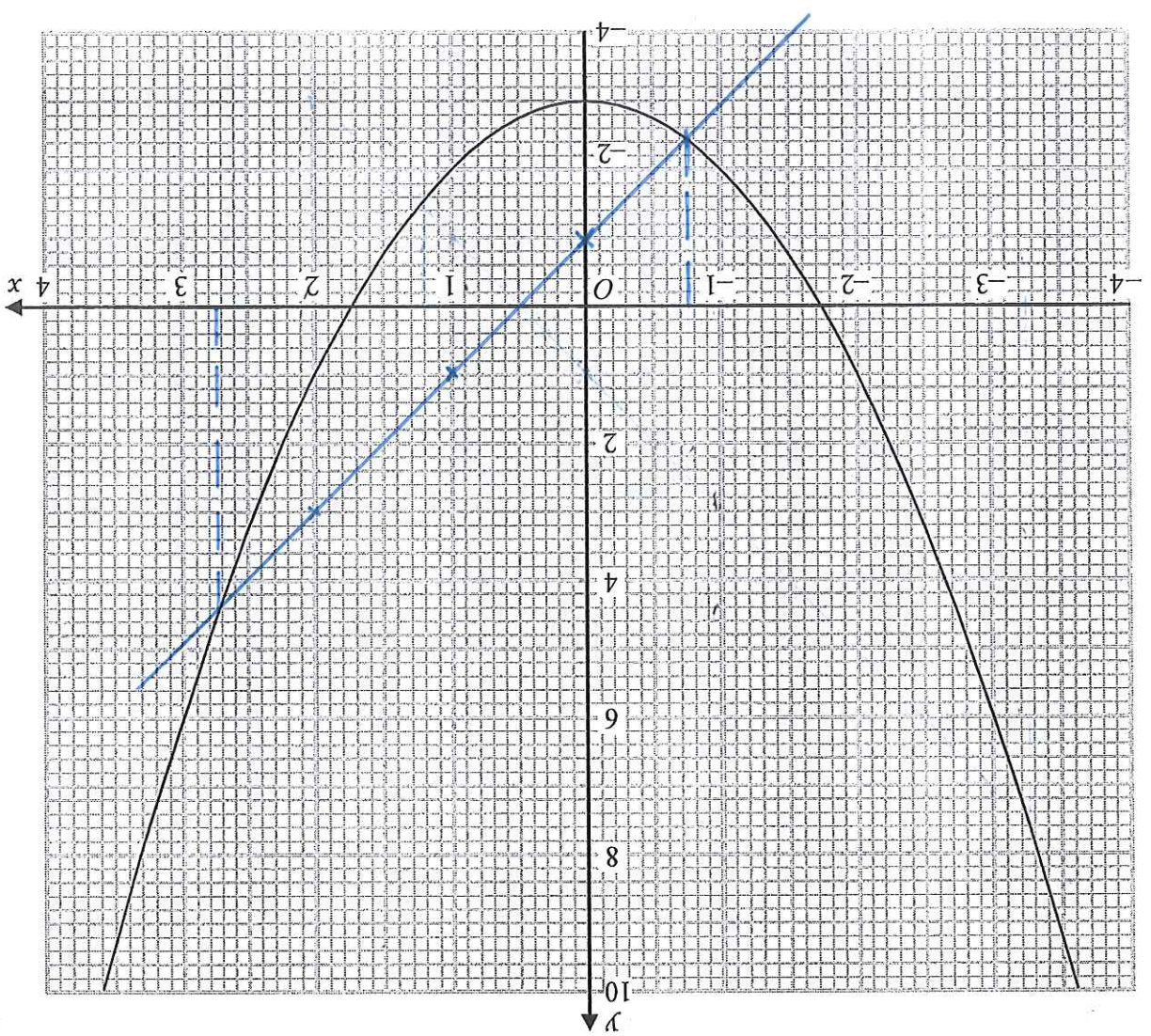
4 is a factor $\therefore$ multiple of 4

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17 Here is the graph of $y = x^2 - 3$



Use the graph to find estimates for the solutions to the equation $x^2 - 2x - 2 = 0$
 You must show how you get your solutions.
 We need to make $x^2 - 2x - 2$ look like $x^2 - 3$

$$x^2 - 2x - 2 = 0 \quad [+2x] \quad x^2 - 2 = 0$$

$$x^2 - 2 = 2x \quad [-1] \quad x^2 - 3 = 2x - 1$$

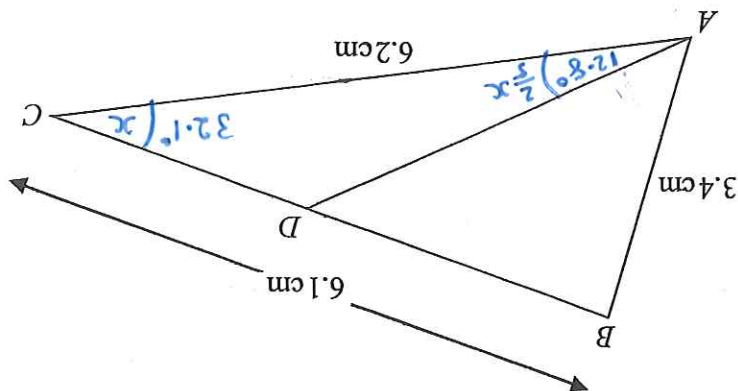
Draw the line $y = 2x - 1$ onto the graph
 The solutions of $x^2 - 2x - 2 = 0$ are the points of
 intersection of $y = x^2 - 3$ and $y = 2x - 1$
 $x \approx -0.75$ and 2.75

(Total for Question 17 is 4 marks)



Turn over

18 The diagram shows triangle ABC.



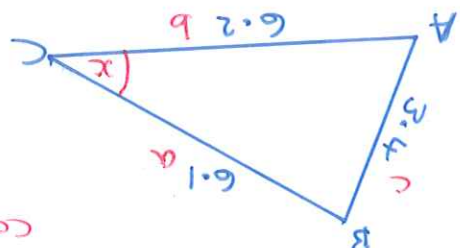
$AB = 3.4 \text{ cm}$ $AC = 6.2 \text{ cm}$ $BC = 6.1 \text{ cm}$

D is the point on BC such that

$$\text{size of angle } DAC = \frac{5}{2} \times \text{size of angle } BCA$$

Calculate the length DC.

Give your answer correct to 3 significant figures.
You must show all your working.



cosine rule to find angle C

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$3.4^2 = 6.1^2 + 6.2^2 - 2 \times 6.1 \times 6.2 \cos C$$

$$-64.09 = -75.64 \cos C$$

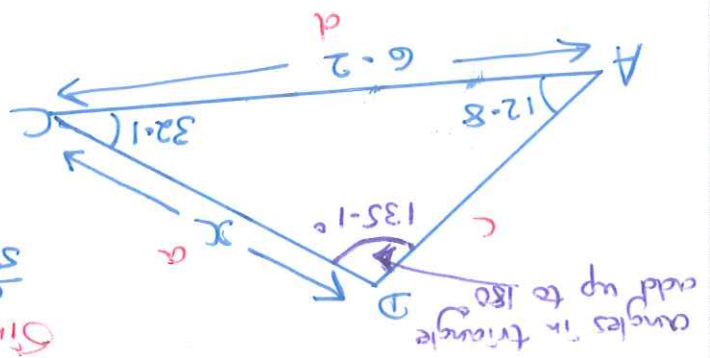
$$0.847303 = \cos C$$

$$C = \cos^{-1}(0.847303)$$

$$= 32.080469^\circ$$

$$= 32.1^\circ \text{ (3sf)}$$

angle $BCA = 32.1^\circ$
 $\therefore \text{angle } DAC = \frac{5}{2} \times 32.080469^\circ$
 $= 12.8321876^\circ$
 $= 12.8^\circ \text{ (3sf)}$



Sine rule to find side DC

$$\frac{a}{\sin A} = \frac{d}{\sin D}$$

$$\frac{32.1}{\sin 12.8} = \frac{d}{\sin 135.1}$$

$$d = \frac{32.1 \times \sin 135.1}{\sin 12.8}$$

$$d = 1.945964$$

$$DC = 1.95 \text{ (3sf)}$$

cm

(Total for Question 18 is 5 marks)

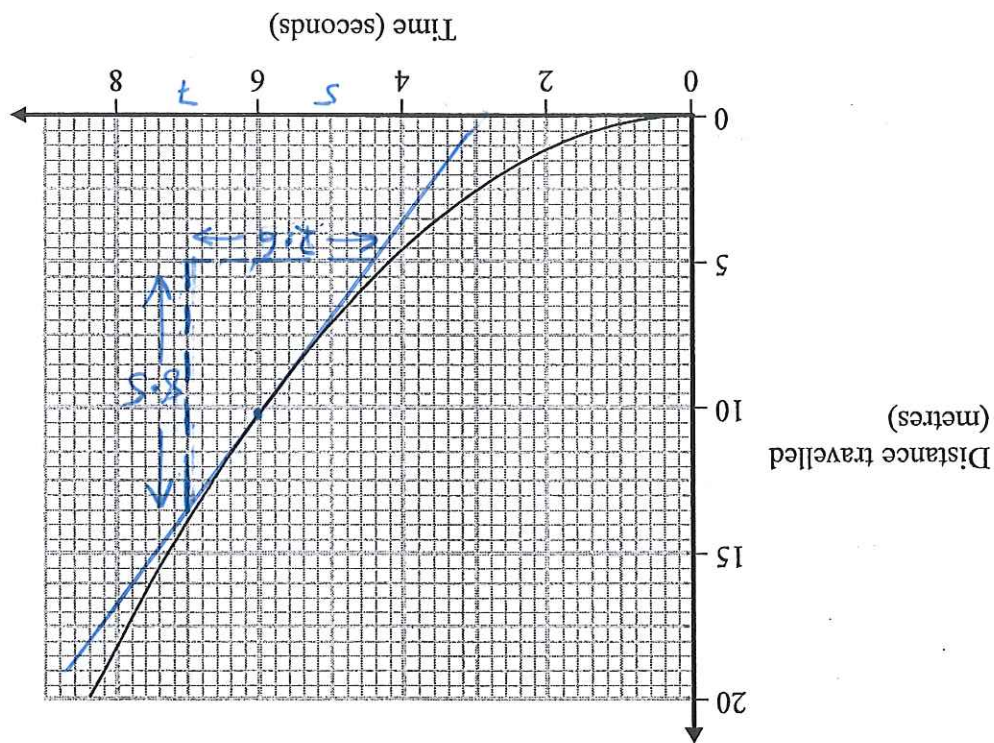


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19 The graph shows information about part of a cyclist's journey.



Work out an estimate of the speed, in m/s, of the cyclist at time 6 seconds.
 This will be the gradient of the tangent to the curve at time 6 seconds

$$\text{speed} \approx \frac{8.5}{2.6} = 3.26923 \text{ m/s}$$

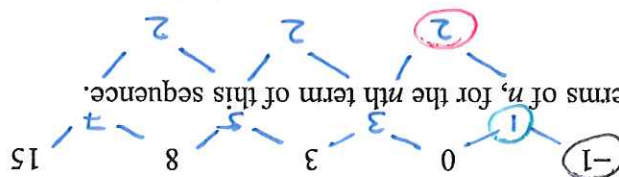
m/s 3.3
 (1dp)
 (Total for Question 19 is 3 marks)





P 5 8 8 7 6 R A 0 1 8 2 4

20 Here are the first five terms of a sequence.



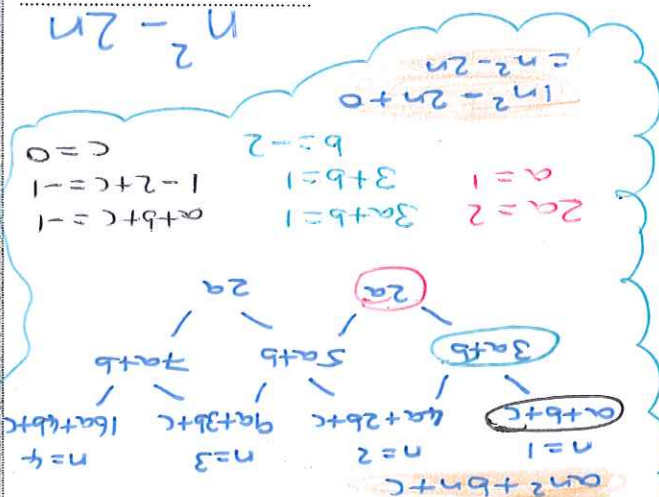
Find an expression, in terms of n , for the n th term of this sequence.

Method 1

n	1	2	3	4	n
n^2	1	4	9	16	n^2
$-2n$	-2	-4	-6	-8	$-2n$
$n^2 - 2n$	-1	0	3	8	$n^2 - 2n$

2nd difference constant $\therefore$ quadratic with n^2

Method 2 (method with method)



(Total for Question 20 is 2 marks)

21 When a biased coin is thrown 4 times, the probability of getting 4 heads is $\frac{16}{81}$

Work out the probability of getting 4 tails when the coin is thrown 4 times.

$$P(4 \text{ heads}) = P(H) \times P(H) \times P(H) \times P(H) = [P(H)]^4$$

$$\therefore [P(H)]^4 = \frac{16}{81}$$

$$P(H) = \sqrt[4]{\frac{16}{81}}$$

$$P(H) = \frac{2}{3}$$

$$\therefore P(T) = \frac{1}{3}$$

(Total for Question 21 is 2 marks)

$$\frac{1}{81}$$

$$= \frac{1}{81} = \left(\frac{1}{3}\right)^4$$

$$P(4 \text{ tails}) = P(T) \times P(T) \times P(T) \times P(T)$$

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22 Show that $\frac{7x-14}{x^2+4x-12} \div \frac{x^3-36x}{x-6}$ simplifies to ax where a is an integer.

$$= \frac{\cancel{7}(x-2)}{\cancel{(x-2)}(x+6)} \div \frac{x-6}{x(x^2-36)}$$

$$= \frac{7}{x+6} \div \frac{\cancel{x-6}}{\cancel{x}(x+6)(x-6)}$$

$$= \frac{7}{x+6} \div \frac{1}{x(x+6)}$$

$$= \frac{7}{\cancel{x+6}} \times \frac{1}{x(\cancel{x+6})}$$

$$= \frac{7x}{1}$$

$$a = 7$$

(Total for Question 22 is 4 marks)

KFC

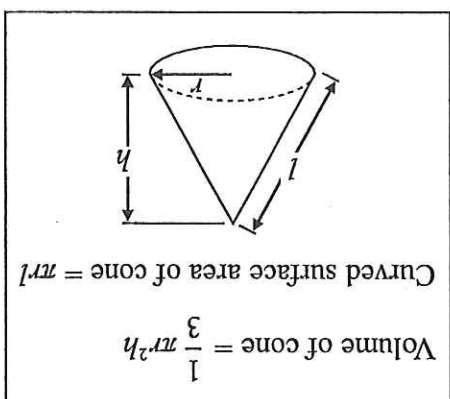
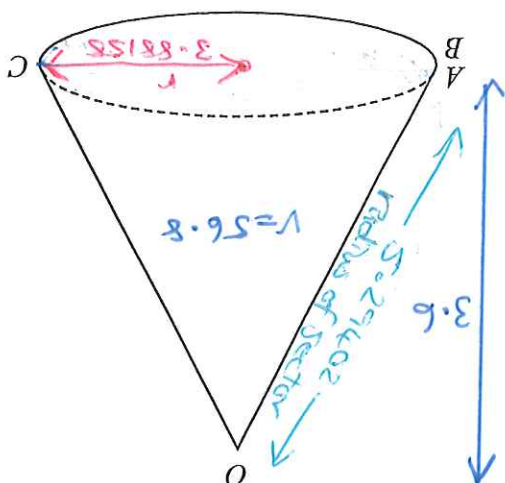
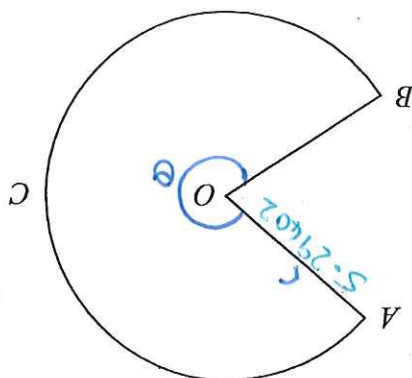


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P 5 8 8 7 6 R A 0 2 0 2 4

- 23 The diagram shows a sector $OACB$ of a circle with centre O . The point C is the midpoint of the arc AB . The diagram also shows a hollow cone with vertex O . The cone is formed by joining OA and OB .



The cone has volume 56.8 cm^3 and height 3.6 cm . Calculate the size of angle AOB of sector $OACB$. Give your answer correct to 3 significant figures. You must show all your working.

Step 1

$$\text{Volume Cone} = 56.8$$

$$\frac{\pi r^2 h}{3} = 56.8$$

$$\pi r^2 (3.6) = 170.4$$

$$r^2 = 15.0666\dots$$

$$r = 3.88158 \text{ cm}$$

Step 3

Curved surface area of cone

$$= \pi r l$$

$$= \pi \times 3.88158 \times 5.29402$$

$$= 64.55711 \text{ cm}^2$$

Curved surface area of cone = area sector

Step 4

Area sector = $\frac{\theta}{360} \times \pi r^2$

$$64.55711 = \frac{\theta}{360} \times \pi \times 5.29402^2$$

$$\theta = 263.95238$$

Step 2

AO is the radius of the sector

$$AO = \sqrt{3.6^2 + 3.88158^2}$$

$$= 5.29402$$

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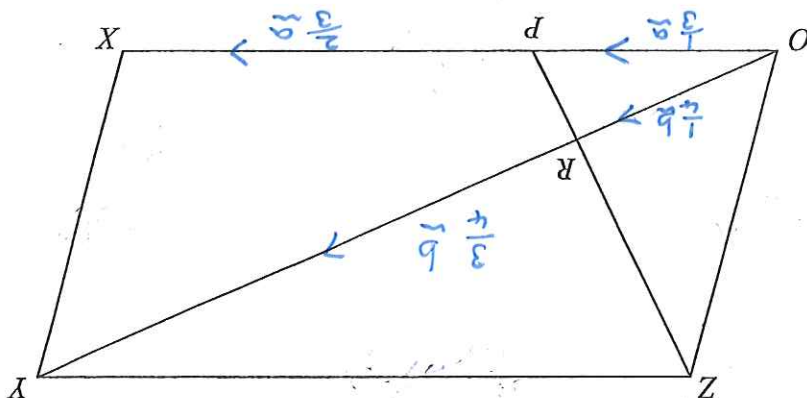
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(Total for Question 23 is 5 marks)

264

24 OXYZ is a parallelogram.



$$\vec{OX} = \mathbf{a}$$

$$\vec{OY} = \mathbf{b}$$

P is the point on OX such that $OP:PX = 1:2$
 R is the point on OY such that $OR:RY = 1:3$

Work out, in its simplest form, the ratio $ZP:ZR$
 You must show all your working.

$$\vec{XY} = -\mathbf{a} + \mathbf{b}$$

$$\vec{OZ} = -\mathbf{a} + \mathbf{b}$$

$$\vec{ZP} = \vec{ZO} + \vec{OP}$$

$$= \mathbf{a} - \mathbf{b} + \frac{1}{3}\mathbf{a}$$

$$= \frac{4}{3}\mathbf{a} - \mathbf{b}$$

$$\vec{ZR} = \vec{ZO} + \vec{OR}$$

$$= \mathbf{a} - \mathbf{b} + \frac{1}{4}\mathbf{b}$$

$$= \mathbf{a} - \frac{3}{4}\mathbf{b}$$

$$ZP : ZR$$

$$\frac{4}{3}\mathbf{a} - \mathbf{b} : \mathbf{a} - \frac{3}{4}\mathbf{b}$$

$$(16\mathbf{a} - 12\mathbf{b}) : (12\mathbf{a} - 9\mathbf{b})$$

$$\times \frac{1}{3}$$

$$1 : \frac{4}{3}$$

$$4 : 3$$

$$4 : 3$$

(Total for Question 24 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS



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