Surname LH

Other name Worked Solutions

Candidate number

Subject

Mathematics

Tier HIGHER ✓

Paper 1H Year 11

21st February 2022

Time: 1 hour 30 minutes

+10% 1h 39 mins +25% 1h 53 mins

Instructions

- Use **black** ink or ball-point pen.
- Answer **all** questions.
- Answer the questions in the spaces provided
- **Calculators must not be used**

Information

- There are 22 questions on this paper
- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Show all of your working out.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium $= \frac{1}{2}(a+b)h$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

Circumference of a circle $= 2\pi r = \pi d$

Area of a circle $= \pi r^2$

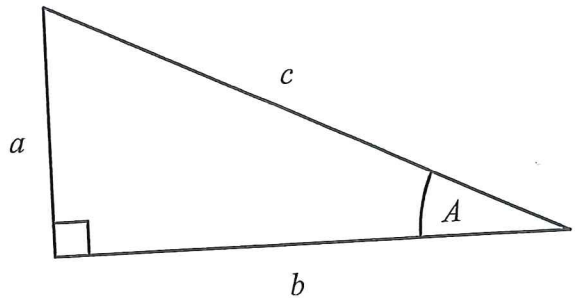
Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

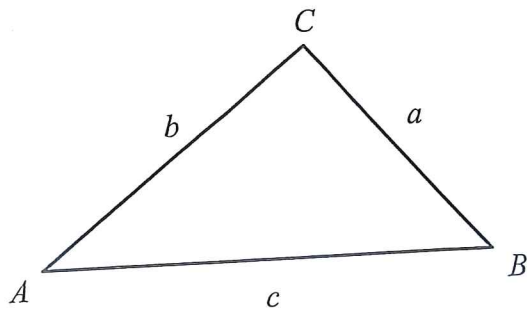


In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b}$$



In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A given B) P(B)

END OF EXAM AID

F Tier
Q 20

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Work out 3.67×4.2

$$\begin{array}{r} 367 \\ \times\ 42 \\ \hline 734 \\ 14680 \\ \hline 15414 \\ \hline 11 \end{array}$$

15.414

(3)

(b) Work out $59.84 \div 1.6$

$$= \frac{59.84}{1.6} \xrightarrow{\times 10} = \frac{598.4}{16}$$

$$\begin{array}{r} 37.4 \\ 16\overline{)598.4} \\ 48 \\ \hline 118 \\ 112 \\ \hline 64 \\ 64 \\ \hline \end{array}$$

37.4

(3)

(Total for Question 1 is 6 marks)

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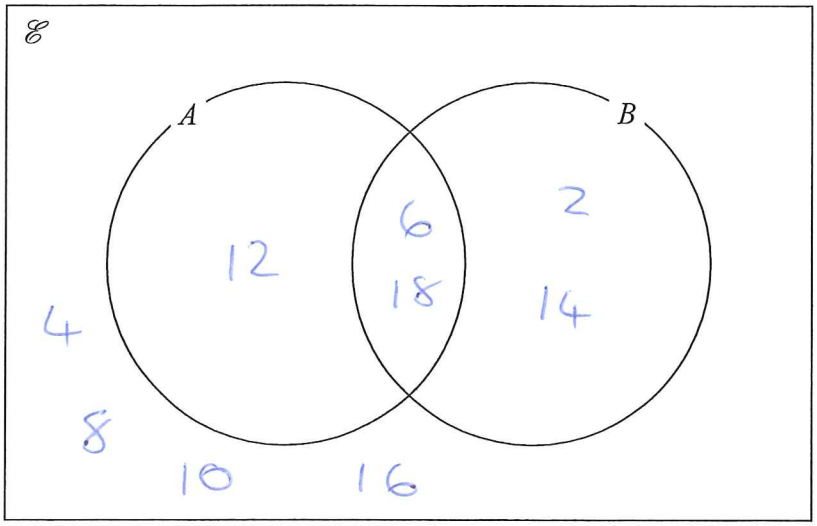
F Tier Q21

2 $\mathscr{E}$ = {even numbers less than 19} = {2, 4, 6, 8, 10, 12, 14, 16, 18}
A = {6, 12, 18}
B = {2, 6, 14, 18}

Complete the Venn diagram for this information.



(Total for Question 2 is 3 marks)

F Tier Q22

3 Work out $4\frac{1}{5} - 2\frac{2}{3}$ $= \frac{21}{5} \times 3 - \frac{8}{3} \times 5$

Give your answer as a mixed number.

$= \frac{63}{15} - \frac{40}{15}$

$= \frac{23}{15}$

$= 1\frac{8}{15}$

$1\frac{8}{15}$

(Total for Question 3 is 3 marks)

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F Tier Q23

4 At the end of 2017
the value of Tamara's house was £220 000
the value of Rahim's house was £160 000

At the end of 2019
the value of Tamara's house had decreased by 20%
the value of Rahim's house had increased by 30%

At the end of 2019, whose house had the greater value?
You must show how you get your answer.

Tamara

£220 000 → 10% £22000, 20% £44000

£220 000
− £ 44 000
£176 000

Rahim

£160 000 → 10% £16000, 30% £48000

£160 000
+ £ 48 000
£208 000

At end of 2019 Tamara's house £176 000
Rahim's how £208 000

Rahim's house had the greater value at the end of 2019

(Total for Question 4 is 4 marks)

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F Tier Q 24

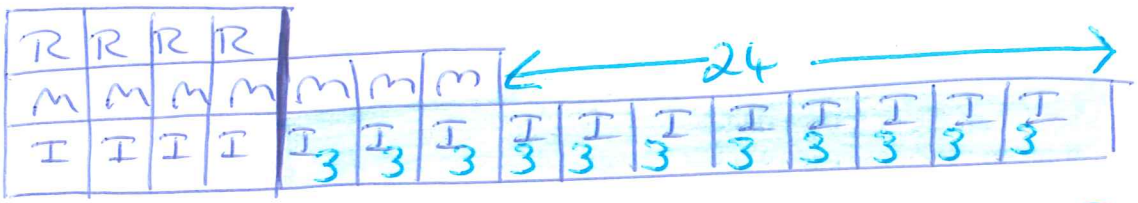
5 Rosie, Matilda and Ibrahim collect stickers.

number of stickers Rosie has : number of stickers Matilda has : number of stickers Ibrahim has = 4 : 7 : 15

R : M : I

Ibrahim has 24 more stickers than Matilda.

Ibrahim has more stickers than Rosie.
How many more?



24 ÷ 8 = 3

11 x 3 = 33

Ibrahim has 33 more stickers than Rosie

33

(Total for Question 5 is 3 marks)

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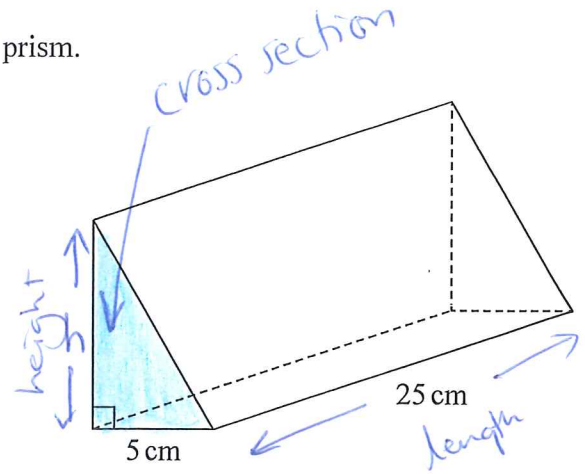
F Tier Q25

6 The diagram shows a prism.



The cross section of the prism is a right-angled triangle.
The base of the triangle has length 5 cm

The prism has length 25 cm
The prism has volume 750 cm^3

Work out the height of the prism.

Volume of Prism = Area cross section x length

$750 = \text{Area triangle} \times 25$ [÷25

$\frac{750}{25} = \text{Area triangle}$

$30 = \text{Area triangle}$

The triangle has area 30 cm^2

$\frac{\text{base} \times \text{height}}{2} = 30$

$\frac{5h}{2} = 30$ [×2

$5h = 60$ [÷5

$h = 12$

12 cm

(Total for Question 6 is 3 marks)

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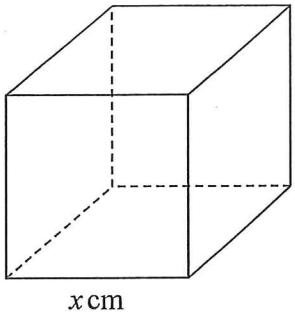
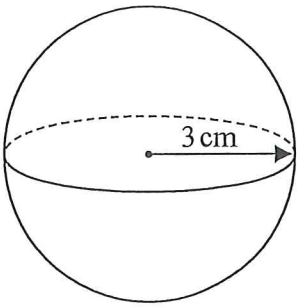
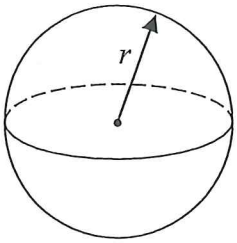
F Tier Q26

7 The diagram shows a cube with edges of length x cm and a sphere of radius 3 cm.







Surface area of sphere = $4\pi r^2$

The surface area of the cube is equal to the surface area of the sphere.

Show that $x = \sqrt{k\pi}$ where k is an integer.

Surface Area of Cube $= 6 \times x^2$
$= 6x^2$

Surface Area of Sphere $= 4 \times \pi \times 3^2$
$= 36\pi$

$6x^2 = 36\pi$ [÷ 6
$x^2 = 6\pi$ [√
$x = \sqrt{6\pi}$

(Total for Question 7 is 4 marks)

End of Foundation Tier
Paper 1F

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8 Solve $x^2 = 5x + 24$

$x^2 - 5x - 24 = 0$

$(x + 3)(x - 8) = 0$

$x + 3 = 0$ or $x - 8 = 0$

$x = -3$ $x = 8$

$x = -3, 8$

(Total for Question 8 is 3 marks)

9 (a) Write down the value of 7^0

1

(1)

(b) Find the value of $3^1 \times 3^6 \times 3^{-6}$

$= 3^{1+6-6}$

$= 3^1$

3

(1)

(c) Find the value of 2^{-4}

$= \frac{1}{2^4}$

$= \frac{1}{16}$

$\frac{1}{16}$

(1)

(d) Find the value of $27^{\frac{1}{3}}$

$= \sqrt[3]{27}$

3

(1)

(Total for Question 9 is 4 marks)

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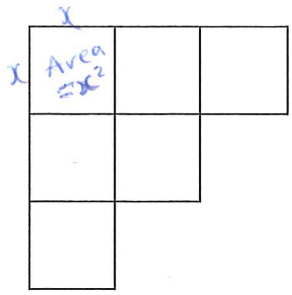
10 The diagram shows a shape made from 6 identical squares.



The total area of the shape is 5406 cm^2

(a) Find an estimate for the length of one side of each square.
Give your answer correct to the nearest whole number.

Area = 6 × Area one square
$= 6 \times x^2$
$= 6x^2$

$6x^2 = 5406$
$6x^2 \approx 5400$ [÷ 6
$x^2 \approx 900$ [√
$x \approx 30$

look for a number divisible by 6 close to 5406
$54 \div 6 = 9$

30 cm

(3)

(b) Is your answer to part (a) an underestimate or an overestimate?
You must give a reason for your answer.

Under-estimate

Area 5406 rounded DOWN

(1)

(Total for Question 10 is 4 marks)

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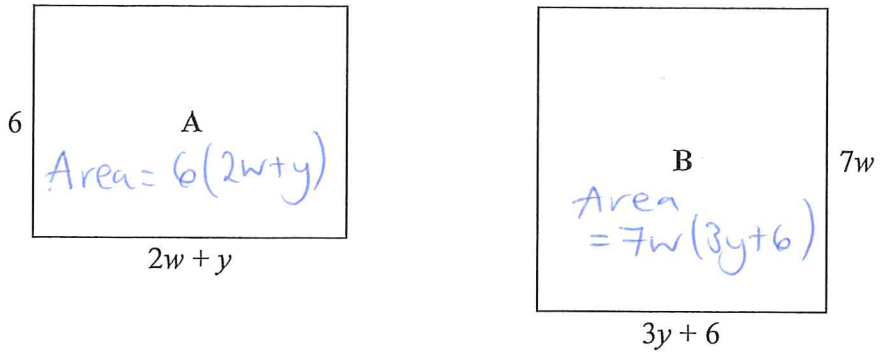
11 The diagram shows two rectangles, A and B.



All measurements are in centimetres.

The area of rectangle A is equal to the area of rectangle B.

Find an expression for y in terms of w.

y is the subject

$6(2w+y) = 7w(3y+6)$

$12w+6y = 21wy + 42w$ [−12w

$6y = 21wy + 30w$ [−21wy

$6y - 21wy = 30w$ [factorise

$3y(2-7w) = 30w$ [÷3

we need y to appear on its own

$y(2-7w) = 10w$ [÷(2−7w)

$y = \frac{10w}{2-7w}$

$y = \frac{10w}{2-7w}$

OR

$6y - 21wy = 30w$

$y(6-21w) = 30w$

$y = \frac{30w}{6-21w}$

(Total for Question 11 is 4 marks)

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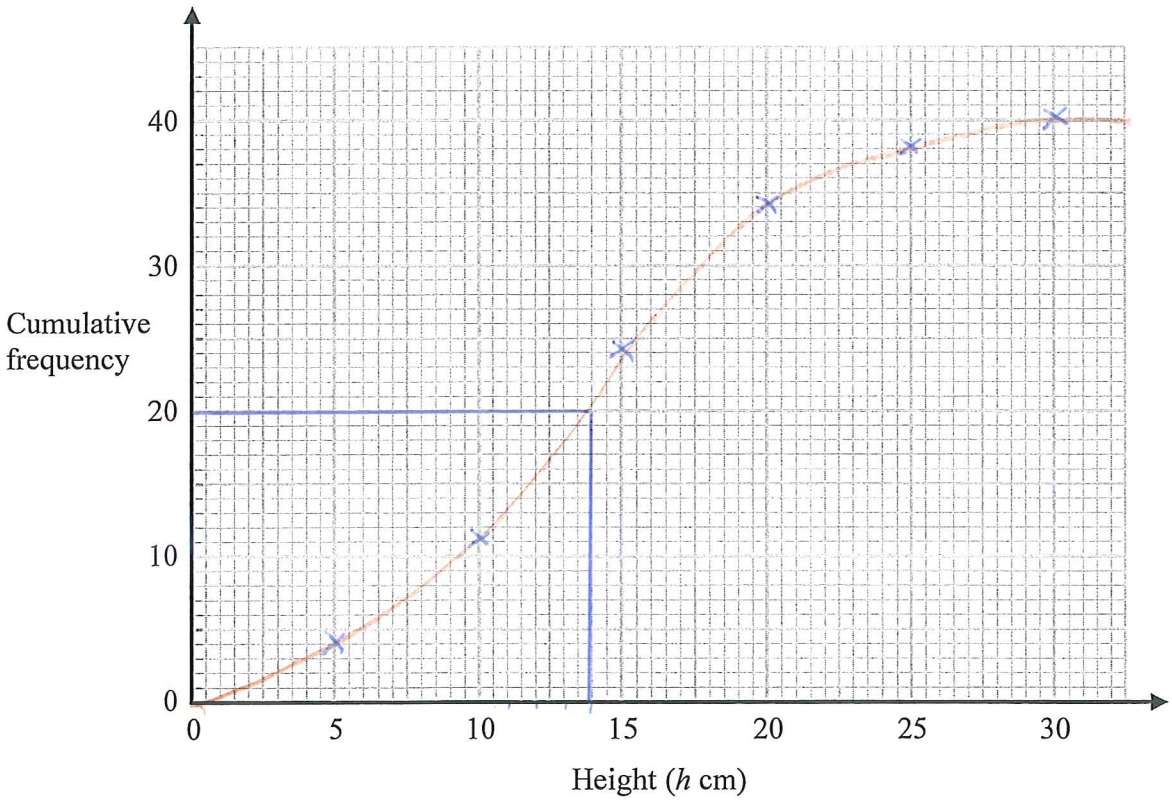
12 The cumulative frequency table gives information about the heights, in cm, of 40 plants.

Height (h cm)	Cumulative Frequency
$0 < h \leqslant 5$	4
$0 < h \leqslant 10$	11
$0 < h \leqslant 15$	24
$0 < h \leqslant 20$	34
$0 < h \leqslant 25$	38
$0 < h \leqslant 30$	40

(a) On the grid, draw a cumulative frequency graph for this information.



(2)

(b) Use the graph to find an estimate for the median height of the plants.

14 cm

allow 13–14

(1)

(Total for Question 12 is 3 marks)

13 Ted is trying to change $0.\dot{4}\dot{3}$ to a fraction.

Here is the start of his method.

$x = 0.\dot{4}\dot{3}$

$10x = 4.\dot{3}\dot{4}$

$10x - x = 4.\dot{3}\dot{4} - 0.\dot{4}\dot{3}$

$9x = 4.343434$
$- \ 0.434343$

Evaluate Ted's method so far.

Ted has multiplied by 10. Because two digits recur he should have multiplied by 100.

When he subtracts to get $9x$ the digits do not line up so he will still have a recurring decimal.

(Total for Question 13 is 1 mark)

not needed for marks, but here is the correct method!

$x = 0.434343\ldots$

$100x = 43.434343\ldots$

$100x = 43.434343\ldots$
$- \ x \quad = 0.434343\ldots$

$99x = 43$

$x = \frac{43}{99}$

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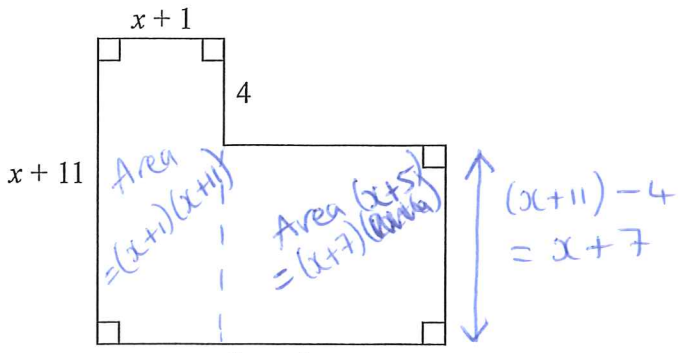
14 Here is a shape with all its measurements in centimetres.



The area of the shape is A cm²

Show that $A = 2x^2 + 24x + 46$

Area $= (x+1)(x+11)$

Area $= (x+7)(x+5)$

$(x+11) - 4 = x + 7$

$(2x+6) - (x+1)$
$= 2x + 6 - x - 1$
$= x + 5$

$A = (x+1)(x+11) + (x+7)(x+5)$
$= x^2 + 11x + x + 11 + x^2 + 5x + 7x + 35$
$= 2x^2 + 24x + 46$

(Total for Question 14 is 3 marks)

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15 Show that $\frac{4x+3}{2x} + \frac{3}{5}$ can be written in the form $\frac{ax+b}{cx}$ where a, b and c are integers.

$\frac{4x+3}{2x} \times \frac{5}{5} + \frac{3}{5} \times \frac{2x}{2x}$

$= \frac{5(4x+3)}{10x} + \frac{6x}{10x}$

$= \frac{20x + 15 + 6x}{10x}$

$= \frac{26x + 15}{10x}$

$a = 26, b = 15, c = 10$

(Total for Question 15 is 3 marks)

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16 There are only 3 red counters and 5 yellow counters in a bag.

Jude takes at random 3 counters from the bag.

Work out the probability that he takes exactly one red counter.

R R R
Y Y Y
Y Y

possibilities

R Y Y
Y R Y
Y Y R

42
× 8
336

$P(R\ Y\ Y) = \frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} = \frac{60}{336}$

$P(Y\ R\ Y) = \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} = \frac{60}{336}$

$P(Y\ Y\ R) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{60}{336}$ +

$\frac{180}{336}$

or $\frac{90}{168}$

or $\frac{45}{84}$

or $\frac{15}{28}$

.....

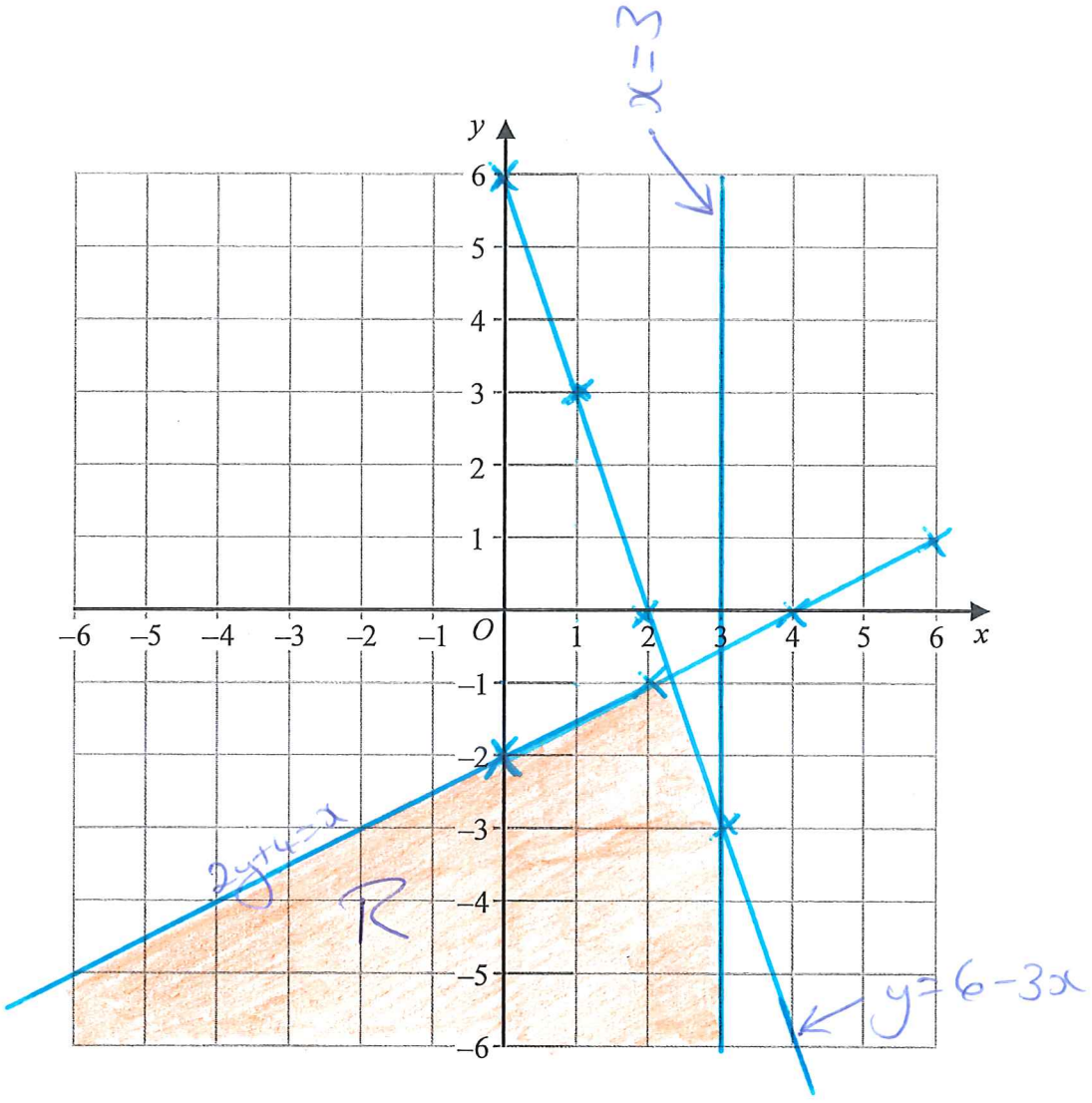
(Total for Question 16 is 4 marks)

17 On the grid show, by shading, the region that satisfies all of these inequalities.

$2y + 4 < x$ $\quad x < 3$ $\quad y < 6 - 3x$

Label the region **R**.

Draw $2y = x - 4$ $\quad y = \frac{1}{2}x - 2$ $\quad x = 3$ $\quad y = -3x + 6$



(Total for Question 17 is 3 marks)

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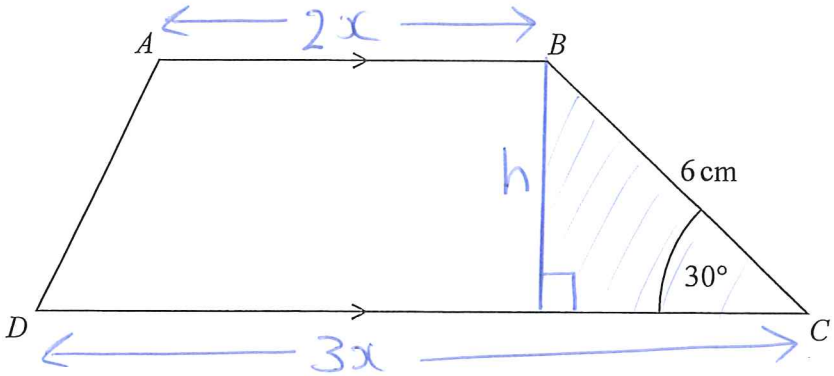
18 Here is trapezium $ABCD$.

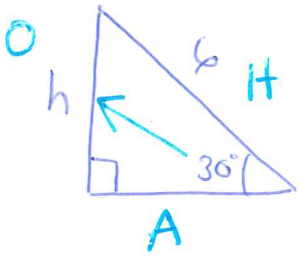


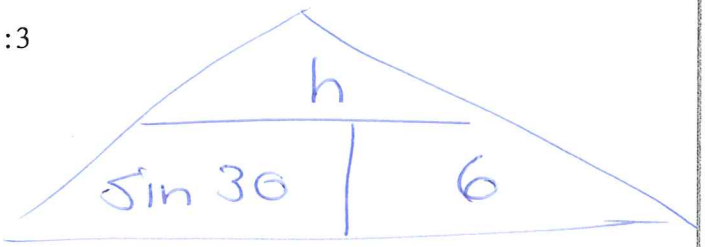
The area of the trapezium is 66 cm²

the length of AB : the length of CD = 2 : 3

Find the length of AB.

O H





$h = \sin(30) \times 6$

$= \frac{1}{2} \times 6$

$= 3$ cm

Area trapezium $= \frac{1}{2}(a+b)h$

$66 = \frac{1}{2}(3x + 2x) \times 3$

$66 = \frac{1}{2}(5x) \times 3$

$66 = \frac{15x}{2}$ [$\times 2$

$132 = 15x$ [$\div 15$

$\frac{132}{15} = x$

15 | 132

19.6 cm

(Total for Question 18 is 5 marks)

15
30
45
60

$8\frac{12}{15} = x$

$8\frac{4}{5} = x$

$AB = 2 \times 8.8$

$= 17.6$ cm

19 Show that $\frac{8+\sqrt{12}}{5+\sqrt{3}}$ can be written in the form $\frac{a+\sqrt{3}}{b}$, where a and b are integers.

$$\frac{8+\sqrt{12}}{5+\sqrt{3}} \times \frac{(5-\sqrt{3})}{(5-\sqrt{3})}$$

$\sqrt{12} = \sqrt{4\times3} = 2\sqrt{3}$

$$= \frac{(8+\sqrt{12})(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})}$$

$$= \frac{40-8\sqrt{3}+5\sqrt{12}-\sqrt{36}}{25-3}$$

$$= \frac{40-8\sqrt{3}+5\times2\sqrt{3}-6}{22}$$

$$= \frac{34-8\sqrt{3}+10\sqrt{3}}{22}$$

$$= \frac{34+2\sqrt{3}}{22} \quad [\div 2$$

(Total for Question 19 is 4 marks)

$$= \frac{17+\sqrt{3}}{11}$$

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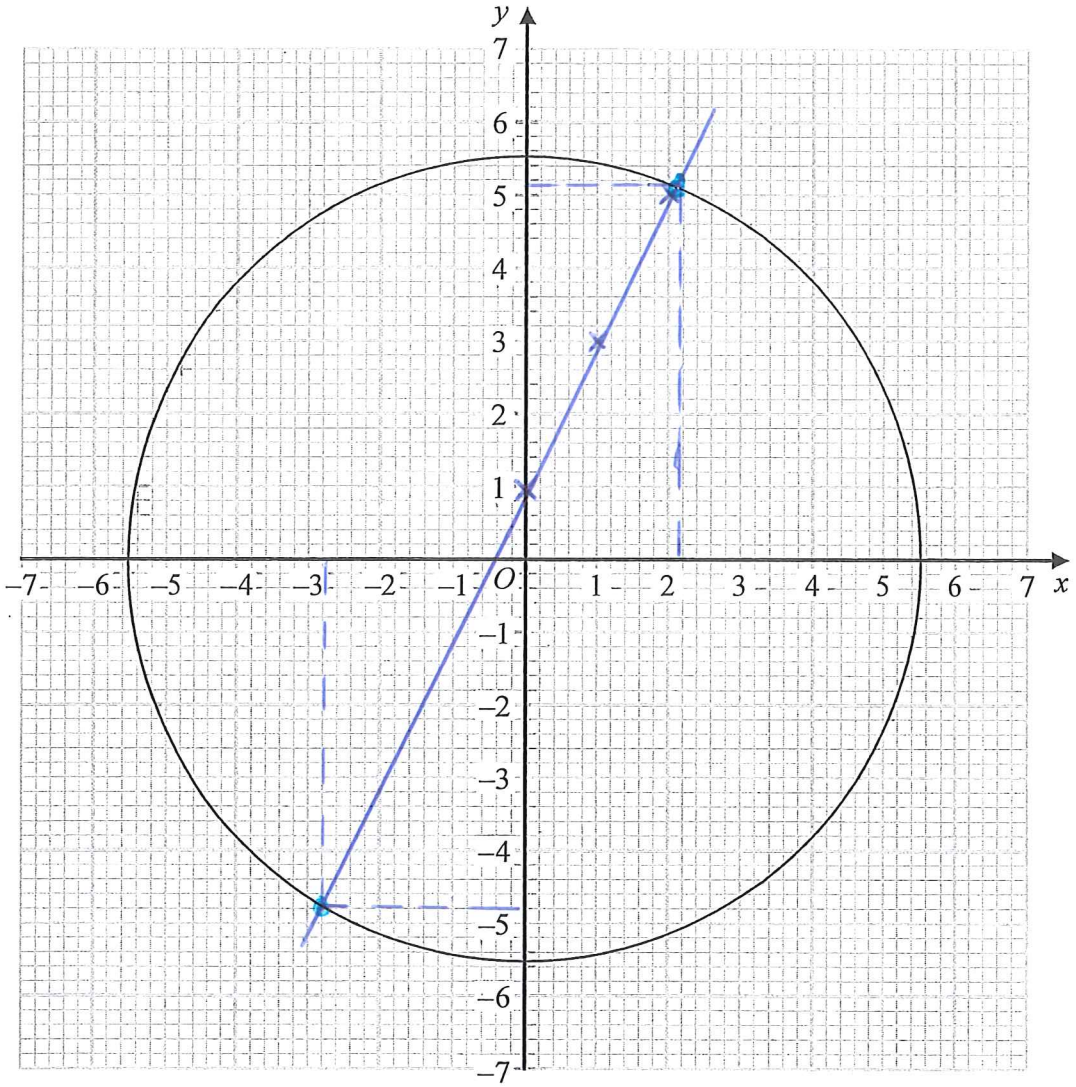
20 The diagram shows the graph of $x^2 + y^2 = 30.25$



Use the graph to find estimates for the solutions of the simultaneous equations

$x^2 + y^2 = 30.25$
$y - 2x = 1$

$y = 2x + 1$

$\{(-2.8, -4.8)\quad (2.2, 5.2)\}$

(Total for Question 20 is 3 marks)

21 The functions f and g are such that

$$f(x) = 3x^2 + 1 \quad \text{for } x > 0 \qquad \text{and} \qquad g(x) = \frac{4}{x^2} \quad \text{for } x > 0$$

(a) Work out gf(1)

$f(1) = 3(1)^2 + 1$
$= 3 + 1$
$= 4$

$g(4) = \frac{4}{4^2} = \frac{4}{16}$

$\frac{1}{4}$

(2)

The function h is such that $h = (fg)^{-1}$

(b) Find h(x)

$fg = f[g(x)]$
$= 3\left(\frac{4}{x^2}\right)^2 + 1$
$= 3\left(\frac{16}{x^4}\right) + 1$
$= 48x^{-4} + 1$
$= \frac{48}{x^4} + 1$

let $y = \frac{48}{x^4} + 1$

swap x and y

$x = \frac{48}{y^4} + 1$

[$\times y^4$

rearrange to make y the subject

[$-y^4$

$xy^4 = 48 + y^4$

[factorise

$xy^4 - y^4 = 48$

[$\div (x-1)$

$y^4(x-1) = 48$

[$\sqrt[4]{}$

$y^4 = \frac{48}{x-1}$

(4)

(Total for Question 21 is 6 marks)

$y = \sqrt[4]{\frac{48}{x-1}}$

$\therefore h(x) = \sqrt[4]{\frac{48}{x-1}}$

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22 Find the coordinates of the turning point on the curve with equation $y = 9 + 18x - 3x^2$
You must show all your working.

$y = -3x^2 + 18x + 9$

$y = -3(x^2 - 6x - 3)$

$= -3\left[(x-3)^2 - 9 - 3\right]$

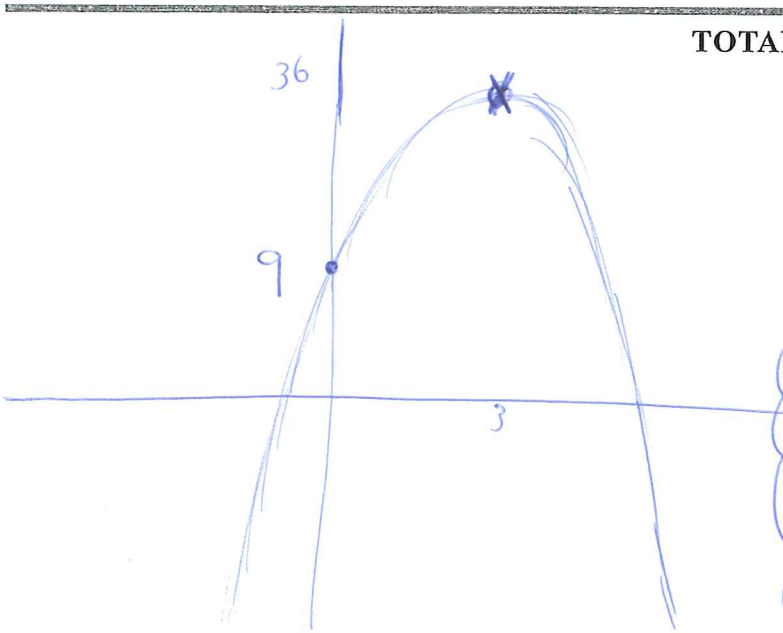
$= -3\left[(x-3)^2 - 12\right]$

$= -3(x-3)^2 + 36$

(3 , 36)

(Total for Question 22 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS



OR, FM students

$y = 9 + 18x - 3x^2$

$\frac{dy}{dx} = 18 - 6x$

at t.p $\frac{dy}{dx} = 0$

$18 - 6x = 0$

$18 = 6x$

$3 = x$

$y = 9 + 18(3) - 3(3)^2$

$= 9 + 54 - 27$

$= 36$