

Surname

LH

Other name

Worked Solution

Candidate number

Subject

Mathematics

Tier HIGHER



Paper 3H Year 11

25th February 2022

Time: 1 hour 30 minutes

+10% 1h 39 mins
+25% 1h 53 mins

Instructions

- Use black ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
- Calculators may be used

Information

- There are 23 questions on this paper
- The total mark for this paper is 80
- The marks for each question are shown in brackets

— use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Show all of your working out.

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2} (a + b) h$$

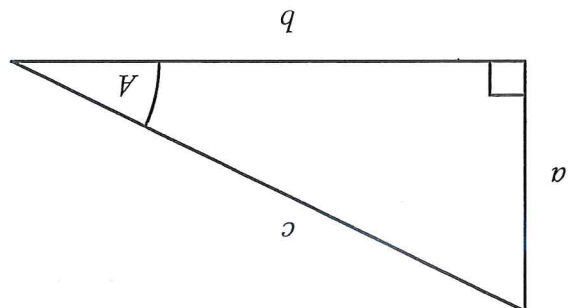
Volume of a prism = area of cross section $\times$ length

Where r is the radius and d is the diameter:

$$\text{Circumference of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$

Pythagoras' Theorem and Trigonometry



$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a , b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b}$$

In any triangle ABC where a , b and c are the length of the sides:

$$\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P \left(1 + \frac{r}{100} \right)^n$$

Probability

Where $P(A)$ is the probability of outcome A and $P(B)$ is the probability of outcome B :

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B) P(B)$$

END OF EXAM AID

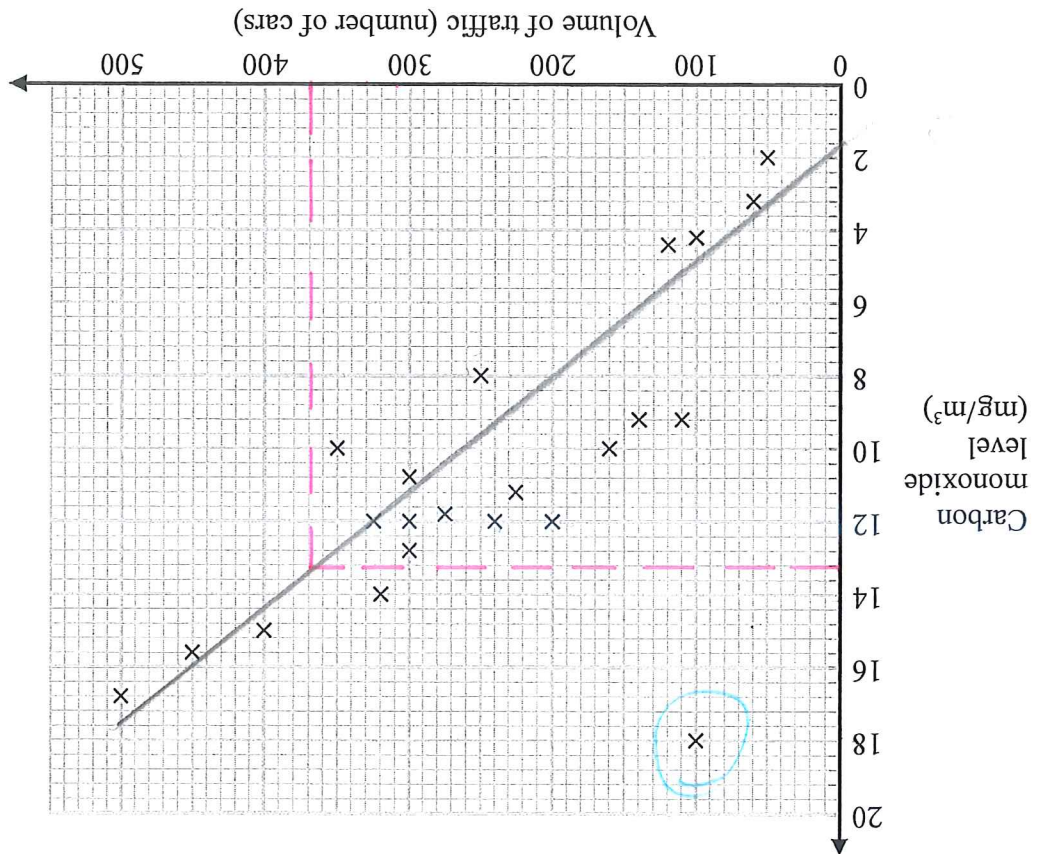
FTIC Q21

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 The scatter graph shows information about the volume of traffic and the carbon monoxide level at a point on a road each day for 22 days.



One point is an outlier.

- (a) Write down the coordinates of this point.

(1) (100, 18)

- For another day, 370 cars pass the point on the road.
(b) Estimate the carbon monoxide level for this day.

Always draw a line of best fit

acceptable range
12.8 → 14.8

(2) 13.2 mg/m³

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Alfie says, "Because there is an outlier, there is no correlation."

(c) Is Alfie correct?

You must give a reason for your answer.

No. There is only one outlier which can be ignored since it does not fit the general trend of the data. It may be incorrectly recorded or due to another factor

(1)

(Total for Question 1 is 4 marks)

2 Natalie makes potato cakes in a restaurant.

She mixes potato, cheese and onion so that

weight of potato : weight of cheese : weight of onion = 9 : 2 : 1

Natalie needs to make 6000 g of potato cakes.

Chese costs £2.25 for 175 g.

Work out the cost of the cheese needed to make 6000 g of potato cakes.

$$9 + 2 + 1 = 12$$

$$6000 \text{ g} \div 12 = 500 \text{ g}$$

Ingredients needed

Potatoes, $9 \times 500 \text{ g} = 4500 \text{ g}$
 Cheese, $2 \times 500 \text{ g} = 1000 \text{ g}$
 Onion, $1 \times 500 \text{ g} = 500 \text{ g}$

Natalie needs 1000g Cheese which is sold in packs of 175g

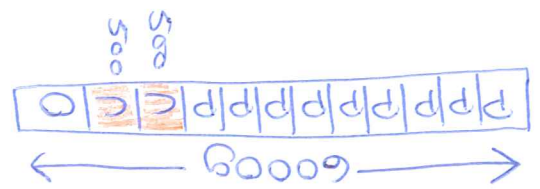
$$1000 \text{ g} \div 175 \text{ g} = 5.714 \dots$$

Natalie will need 6 packs of cheese

$$6 \times £2.25 = £13.50$$

note

ms also accepts answers which do not use complete packs of cheese.
 Accept £12.85 or £12.86



£ 13.50

(Total for Question 2 is 4 marks)

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3 (a) Write 4.5×10^5 as an ordinary number.

450000.
45000.
4500.
450.
45.
(4.5)

(b) Write 0.007 in standard form.

7×10^{-3}

(1)

(c) Work out $4.2 \times 10^3 + 5.3 \times 10^2$.
Give your answer in standard form.

Use calculator to give 4730

4.73×10^3

(2)

(Total for Question 3 is 4 marks)

F Tier 0.23

A water tank is empty. Anil needs to fill the tank with 2400 litres of water.

Company A supplies water at a rate of 8 litres in 1 minute 40 seconds. Company B supplies water at a rate of 2.2 gallons per minute.



Company A would take more time to fill the tank than Company B would take to fill the tank.

How much more time?

Give your answer in minutes correct to the nearest minute.

Company A



8 L : 1 min 40 secs
8 L : $1\frac{40}{60}$ mins
8 L : $1\frac{2}{3}$ mins

$\times 300$

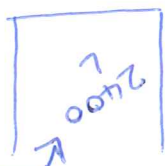
2400 L : 500 mins

Company A takes 500 minutes to fill the tank

500 mins = $8\frac{1}{3}$ hours

or 8 hours 20 mins.

Company B



2.2 gallons : 1 min
9.988 L : 1 min

$\div 9.988$

$\div 9.988$

1 L : $\frac{2497}{250}$ mins

$\times 2400$

2400 L : 240.288 mins

Company B takes 240.288346 minutes to fill the tank

Company A - Company B

500 mins - 240.288346 mins

= 259.711

= 260 minutes (to nearest minute)

(Total for Question 4 is 4 marks)

minutes

260

Careful changing 1 min 40 secs into minutes (it is not 1.4 minutes!)
You could use the time button on your calculator

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F T10 Q25

5 The first four terms of a Fibonacci sequence are

a $2a$ $3a$ $5a$ $8a$

The sum of the first five terms of this sequence is 228

Work out the value of a .

$$a + 2a + 3a + 5a + 8a = 228$$

$$19a = 228$$

$$[\div 19$$

$$a = 12$$

$$a = 12$$

(Total for Question 5 is 3 marks)

F Tier Q26

6 In a bag there are only red counters, blue counters, green counters and pink counters. A counter is going to be taken at random from the bag.

The table shows the probabilities of taking a red counter or a blue counter.

Colour	red	blue	green	pink
Probability	0.05	0.15	0.5	0.3

The probability of taking a green counter is 0.2 more than the probability of taking a pink counter.

(a) Complete the table.

$$0.05 + 0.15 = 0.2$$

$$1 - 0.2 = 0.8$$

$$P(G) + P(P) = 0.8$$

$$\text{Let } P(P) = x$$

$$\text{then } P(G) = x + 0.2$$

There are 18 blue counters in the bag.

(b) Work out the total number of counters in the bag.

15% of the counters are blue
 Let the total number of counters be T
 15% of T = 18 counters
 $0.15 T = 18$
 $T = 120$

(Total for Question 6 is 4 marks)

(2)

120

(2)

$$x + x + 0.2 = 0.8$$

$$2x + 0.2 = 0.8$$

$$2x = 0.6$$

$$x = 0.3$$

$$P(P) = 0.3$$

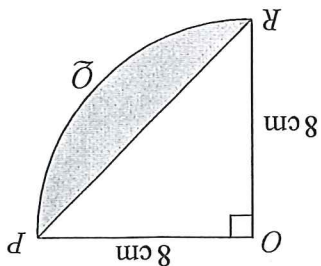
$$P(G) = 0.3 + 0.2 = 0.5$$

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7 The diagram shows a sector $OPQR$ of a circle, centre O and radius 8 cm.



OPR is a triangle.

Work out the area of the shaded segment PQR .

Give your answer correct to 3 significant figures.

$$\text{Area triangle} = \frac{\text{base} \times \text{perpendicular height}}{2}$$

$$= \frac{8 \times 8}{2}$$

$$= \frac{64}{2}$$

$$= 32 \text{ cm}^2$$

$$\text{Area Sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times \pi \times 8^2$$

$$= \frac{1}{4} \times \pi \times 8^2$$

$$= 16\pi \text{ cm}^2$$

$$\text{Shaded Area} = 16\pi - 32$$

$$= 18.265482 \dots$$

$$= 18.3 \text{ cm}^2 \text{ (3sf)}$$

$$18.3 \text{ cm}^2$$

angle at centre, $\theta = 90^\circ$
radius, $r = 8$

Area Shaded segment

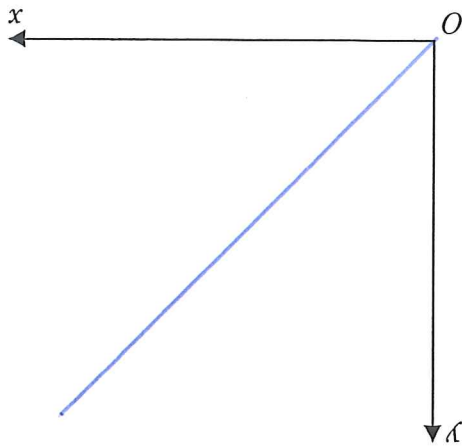
$$= \text{Area Sector} - \text{Area Triangle}$$

End of Foundation Tier Paper

(Total for Question 7 is 4 marks)

8 (a) Using the axes below, sketch a graph to represent the statement

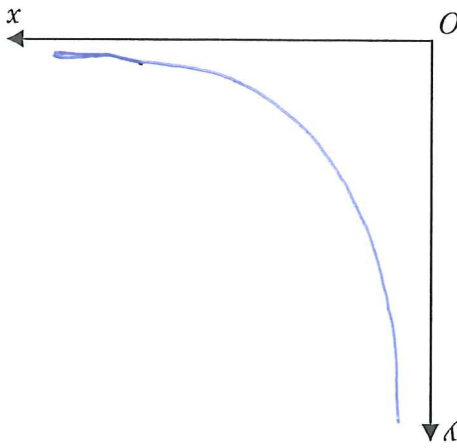
y is directly proportional to x



(1)

(b) Using the axes below, sketch a graph to represent the statement

y is inversely proportional to x



(1)

(Total for Question 8 is 2 marks)

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9

On Monday, 12 people took 5 hours to clean a number of cars.
On Tuesday, 15 people cleaned the same number of cars.

Assuming that all the people worked at the same rate,

(a) work out how many hours the 15 people took to clean the cars.

$$\begin{array}{rcl}
 & \uparrow \div 12 & \\
 12 & : & 5 \\
 \times 15 & \uparrow & \\
 1 & : & 60 \\
 \times 15 & \uparrow & \\
 15 & : & 4
 \end{array}$$

people : hours

people and hours
are inversely
proportional

The assumption is wrong.

(b) How might this affect the time taken for the 15 people to clean the cars?

If the people do not all work at the same
rate then the time could be more or less
than 4 hours.

(1)

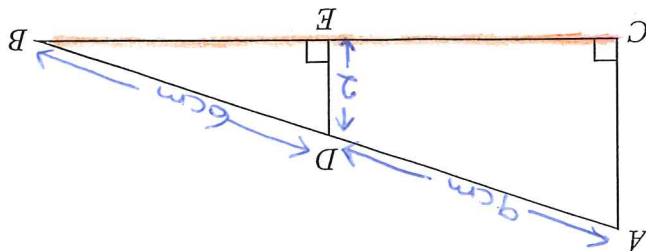
(Total for Question 9 is 3 marks)

(2)

hours

4

10 The diagram shows two right-angled triangles ACB and DEB .

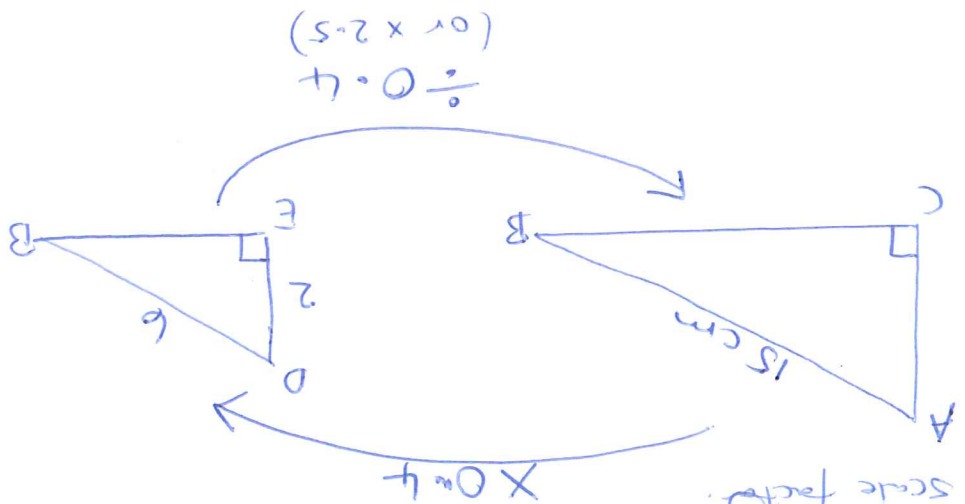


$AD = 9$ cm
 $DE = 2$ cm
 $DB = 6$ cm

Calculate the length of CB .

Give your answer correct to 2 decimal places.

The triangles are similar and therefore we can find a scale factor.



14.14 cm

(Total for Question 10 is 4 marks)

Use Pythagoras' Theorem to find BE

$$BE^2 = 6^2 - 2^2$$

$$= 32$$

$$BE = \sqrt{32}$$

$$= 4\sqrt{2}$$

$$CB = BE \div 0.4$$

$$= 4\sqrt{2} \div 0.4$$

$$= 10\sqrt{2}$$

$$= 14.142135$$

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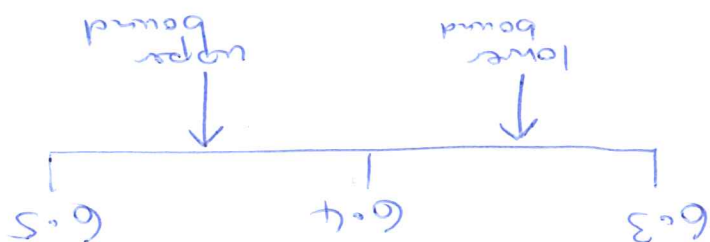
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11 Freya writes down the value of x , correct to 1 decimal place.

She writes $x = 6.4$

Complete the error interval for x .



$$6.35 \leq x < 6.45$$

(Total for Question 11 is 2 marks)

12 $(ax^6)^{\frac{1}{n}} = 7x^3$

Work out the value of a and the value of n .

$$a^{\frac{1}{n}} \times x^{\frac{6}{n}} = 7x^3$$

$$a^{\frac{1}{n}} = 7$$

$$\sqrt[n]{a} = 7$$

$$\sqrt[2]{49} = 7$$

giving $a = 49, n = 2$

$$\frac{6}{n} = 3$$

$$n = 2$$

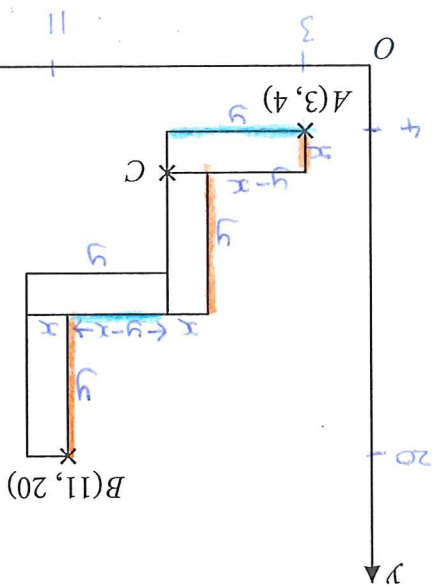
check that these values work

$$a = 49$$

$$n = 2$$

(Total for Question 12 is 2 marks)

13 A pattern is made from four identical rectangles. The sides of the rectangles are parallel to the axes.



Point A has coordinates (3, 4)
Point B has coordinates (11, 20)
Point C is marked on the diagram.
You must show all your working.

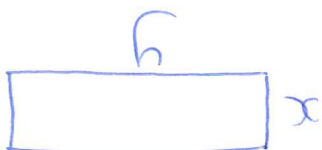
Horizontal
Distance $x = 3 \rightarrow x = 11$
 $y + y - x = 8$
 $2y - x = 8$
Vertical
Distance $y = 4 \rightarrow y = 20$
 $x + y + y = 16$
 $2y + x = 16$

Solve simultaneously

$2y - x = 8$ ①
 $2y + x = 16$ ②

① + ②: $4y = 24$
 $y = 6$

Subs m ②: $2(6) + x = 16$
 $12 + x = 16$
 $x = 4$



To get from point A to point C travel y right and x up
(3, 4)
↑ +6
↑ +4
point C at (9, 8)

16 units

(9, 8)

(Total for Question 13 is 5 marks)

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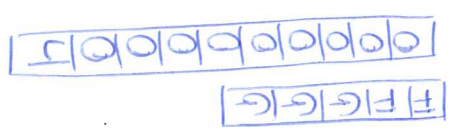
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14 Olivia and Jessica have in total half as many sweets as Fran and Gary have in total.

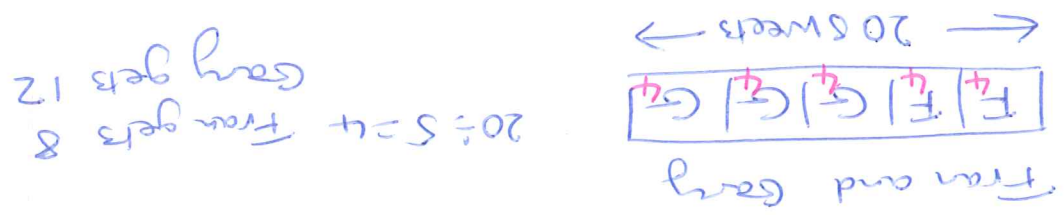
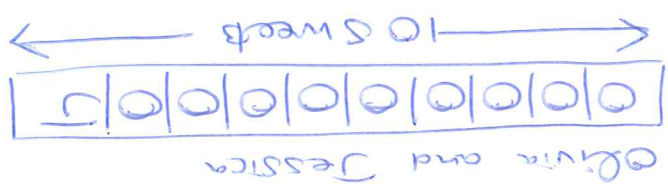
Fran and Gary share their sweets in the ratio 2 : 3
Olivia and Jessica share their sweets in the ratio 9 : 1



Fran got w sweets.
Gary got x sweets.
Olivia got y sweets.
Jessica got z sweets.

Find, in its simplest form, $w : x : y : z$

If Olivia and Jessica have 10 sweets then
Fran and Gary will have 20 sweets



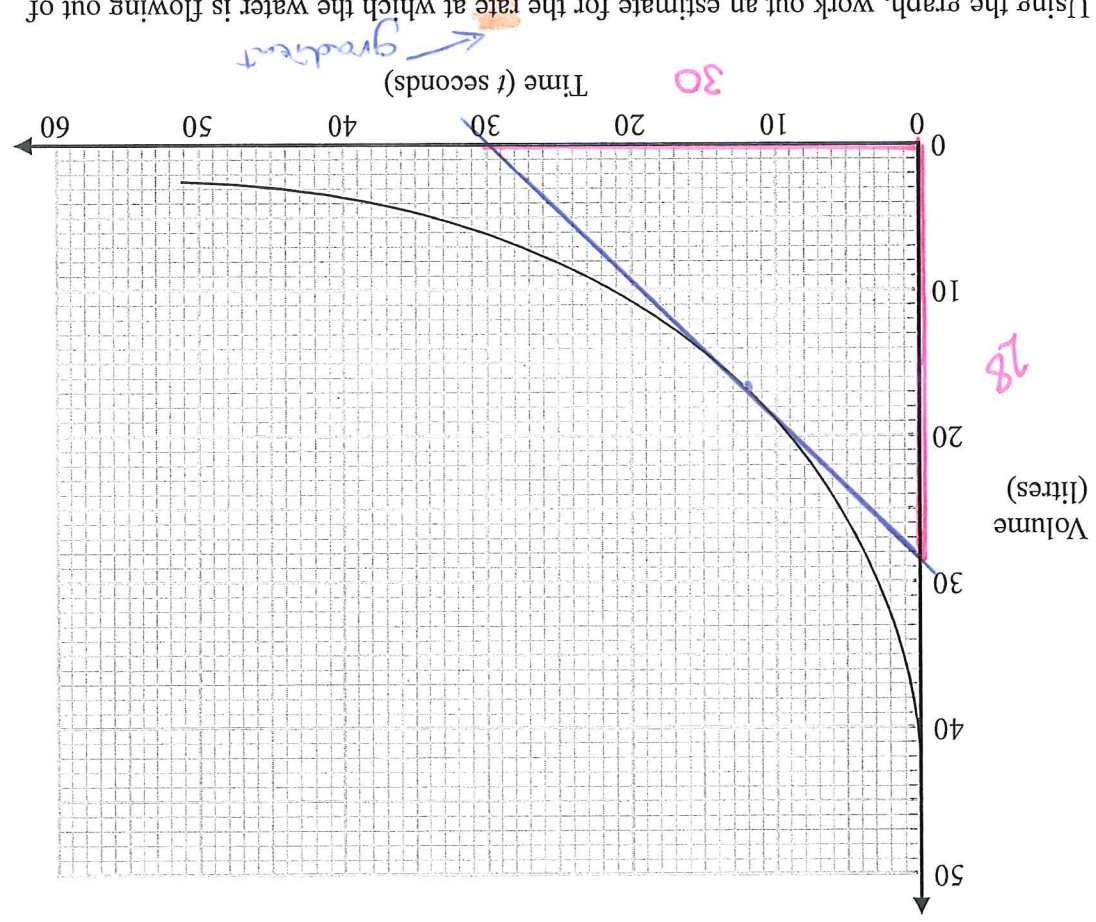
$20 \div 5 = 4$
Fran gets 8
Gary gets 12
 $w = 8$
 $x = 12$

$w : x : y : z$
 $8 : 12 : 9 : 1$

$8 : 12 : 9 : 1$

(Total for Question 14 is 4 marks)

15 The graph gives the volume of water, in litres, in a container at time t seconds after the water started to flow out of the container.



Using the graph, work out an estimate for the rate at which the water is flowing out of the container when $t = 12$

You must show your working.

We need to draw a tangent to the curve at $t = 12$ and find its gradient. The bigger the triangle you use to find the gradient, the more accurate you are likely to be.

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{28}{30}$$

$$0.933$$

litres per second

$$= 0.933 \text{ (3sf)}$$

Accept

$$0.7 \rightarrow 1.1$$

(Total for Question 15 is 3 marks)

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16 The curve C has equation $y = x^2 + 3x - 3$

The line L has equation $y - 5x + 4 = 0$

Show, algebraically, that C and L have exactly one point in common.

$$y - 5x + 4 = 0 \quad \text{rearrange to make } y \text{ the subject}$$

$$y = 5x - 4$$

Equate expressions for y

$$x^2 + 3x - 3 = 5x - 4 \quad [-5x]$$

$$x^2 - 2x - 3 = -4 \quad [+4]$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)(x - 1) = 0$$

only one point of intersection as only one solution

$$x - 1 = 0$$

$$x = 1$$

$$\text{When } x = 1, y = 5(1) - 4$$

$$y = 5 - 4$$

$$y = 1$$

The curve C and line L have one point in common (1, 1)

Solve simultaneously

17 x is directly proportional to the square of y .
 y is directly proportional to the cube of z .

$$z = 2 \text{ when } x = 32$$

Find a formula for x in terms of z .

$$x \propto y^2$$

$$x = ky^2$$

$$y \propto z^3$$

$$y = pz^3$$

where k and p are constants of proportionality

$$x = k \times y^2$$

$$= k \times (pz^3)^2$$

$$= k \times p^2 z^6$$

Let us use C to represent.

$$x = C z^6$$

when $z = 2$, $x = 32$

$$32 = C \times 2^6$$

$$32 = C \times 64$$

$$[\div 64]$$

$$\frac{32}{64} = C$$

$$\frac{1}{2} = C$$

$$\therefore x = \frac{1}{2} z^6$$

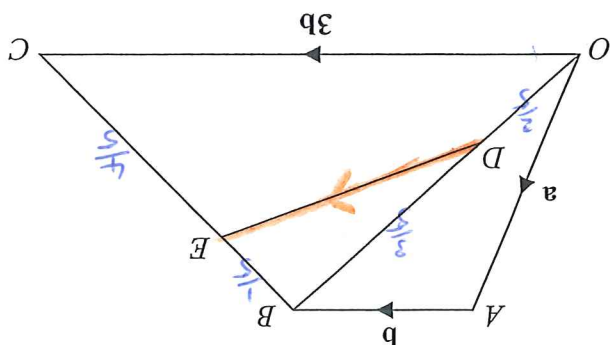
(Total for Question 17 is 4 marks)

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18 $OABC$ is a trapezium.



$$\begin{aligned}\vec{OA} &= \mathbf{a} \\ \vec{AB} &= \mathbf{b} \\ \vec{OC} &= 3\mathbf{b}\end{aligned}$$

D is the point on OA such that $OD:DA = 2:3$
 E is the point on BC such that $BE:EC = 1:4$

Work out the vector $\vec{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
 Give your answer in its simplest form.

$$\begin{aligned}\vec{DE} &= \vec{DB} + \vec{BE} \\ &= \frac{3}{5}\vec{OB} + \frac{1}{5}\vec{BC}\end{aligned}$$

$$\vec{OB} = \vec{a} + \vec{b}$$

$$\begin{aligned}\vec{BC} &= -\vec{b} - \vec{a} + 3\vec{b} \\ &= 2\vec{b} - \vec{a}\end{aligned}$$

$$\begin{aligned}\vec{DE} &= \frac{3}{5}(\vec{a} + \vec{b}) + \frac{1}{5}(2\vec{b} - \vec{a}) \\ &= \frac{3}{5}\vec{a} + \frac{3}{5}\vec{b} + \frac{2}{5}\vec{b} - \frac{1}{5}\vec{a} \\ &= \frac{2}{5}\vec{a} + \frac{5}{5}\vec{b} \\ &= \frac{2}{5}\vec{a} + \vec{b}\end{aligned}$$

(Total for Question 18 is 4 marks)

19 At the start of year n , the number of animals in a population is P_n

At the start of the following year, the number of animals in the population is P_{n+1} where

$$P_{n+1} = kP_n$$

At the start of 2017 the number of animals in the population was 4000
At the start of 2019 the number of animals in the population was 3610

Find the value of the constant k .

$$\begin{aligned} 2017 \quad n=0 \quad P_0 &= 4000 \\ 2018 \quad n=1 \quad P_1 &= k \times 4000 \\ 2019 \quad n=2 \quad P_2 &= k \times 4000k \\ &= 4000k^2 \end{aligned}$$

$$4000k^2 = 3610 \quad [\div 4000]$$

$$k^2 = 0.9025 \quad [\sqrt{\quad}]$$

$$k = 0.95$$

$$0.95$$

(Total for Question 19 is 3 marks)

20 Pat throws a fair coin n times.

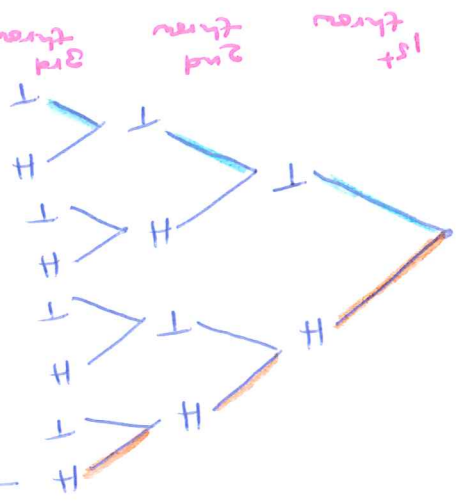
Find an expression, in terms of n , for the probability that Pat gets at least 1 head and at least 1 tail.

$$P(\text{Head}) = \frac{1}{2} \quad P(\text{Tail}) = \frac{1}{2} \quad n \text{ throws}$$

$$\rightarrow P(\text{all heads}) = \left(\frac{1}{2}\right)^n$$

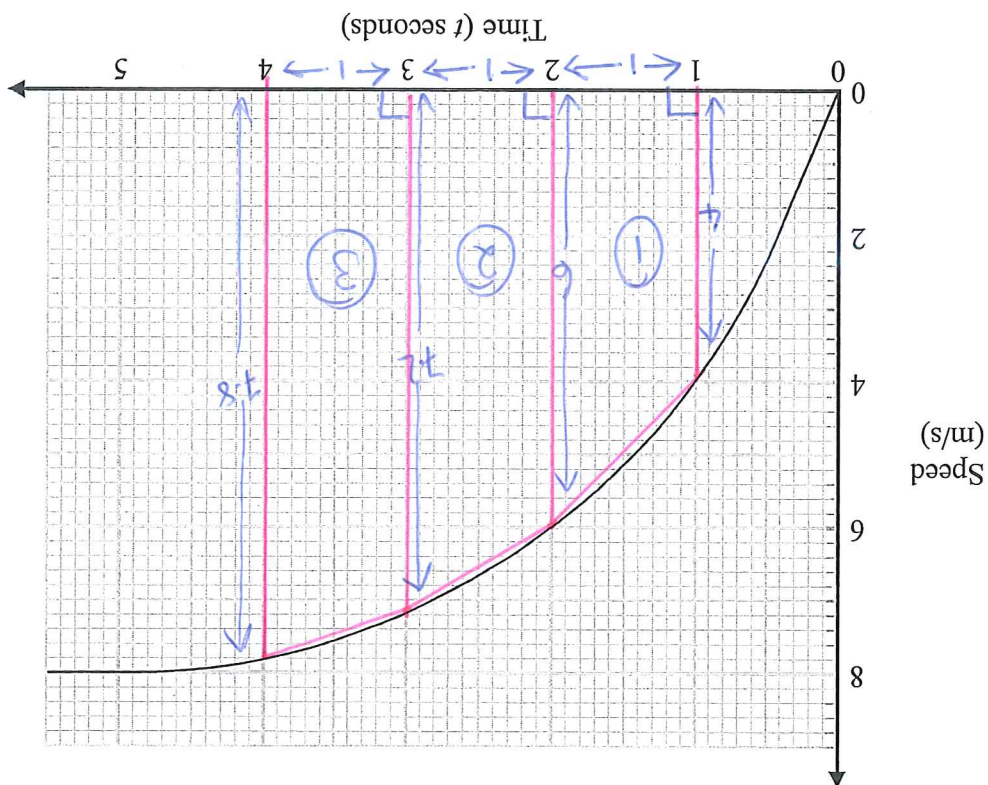
$$\begin{aligned} P(\text{at least one H and at least one T}) &= 1 - [P(\text{all H}) + P(\text{all T})] \\ &= 1 - \left[\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n\right] \end{aligned}$$

$$\rightarrow P(\text{all Tails}) = \left(\frac{1}{2}\right)^n$$



(Total for Question 20 is 2 marks)

21 Here is a speed-time graph showing the speed, in metres per second, of an object t seconds after it started to move from rest.



(a) Using 3 trapeziums of equal width, work out an estimate for the area under the graph between $t = 1$ and $t = 4$

$$\begin{aligned} \text{Area trapezium ①} &= \frac{1}{2}(4+6) \times 1 = 5 \\ \text{Area trapezium ②} &= \frac{1}{2}(6+7.2) \times 1 = 6.6 \\ \text{Area trapezium ③} &= \frac{1}{2}(7.2+7.8) \times 1 = 7.5 \end{aligned}$$

+

19.1

(3)

(b) What does this area represent?

the distance travelled by the object between 1 and 4 seconds

(1)

(Total for Question 21 is 4 marks)

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an integer.

$$\frac{6x^3}{9x^2 - 144}$$

$$= \frac{2x^3}{3(x+4)(x-4)}$$

$$\frac{6x^3(3x+12)(3x-12)}{6x^3(3x+4)(3x-4)}$$

So we have

$$= \frac{3(x+4)(x-4)}{2x^3} \times \frac{2x^4}{3(x-4)} = \frac{(x+4)(x-4)}{2x^3} \times \frac{2x^4}{3(x-4)}$$

$$\frac{(x+4)x}{1} = \frac{\cancel{8x}(x+4)}{\cancel{3}}$$

$$\frac{(1+x)x}{(1+x)8x} = \frac{1}{8}$$

$$= \frac{x(3x+12)}{5}$$

$$= \frac{2x(3x+12)}{6x}$$

$$= \frac{6x^3}{(3x+12)(3x-12)} \times \frac{3x-12}{2x^4}$$

$$= \frac{6x^3(3x+12)(3x-12)}{2x^4(3x-12)}$$

$$\frac{6x^3}{(3x+12)(3x-12)} \cdot \frac{3(x-4)}{2x^2}$$

Alternative

(Total for Question 22 is 3 marks)

