Write your name here

Surname	Other Names

Mathematics
2022 Paper 2 (Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Calculators may be used**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

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Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium $= \frac{1}{2}(a + b)\,h$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

Circumference of a circle $= 2\pi r = \pi d$
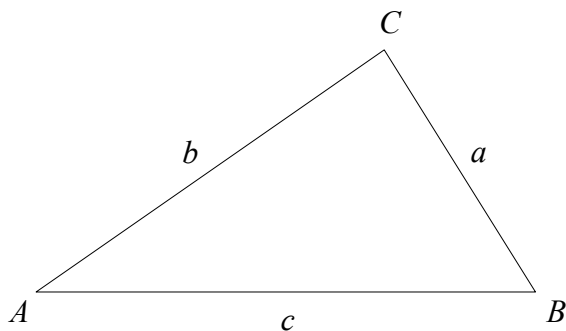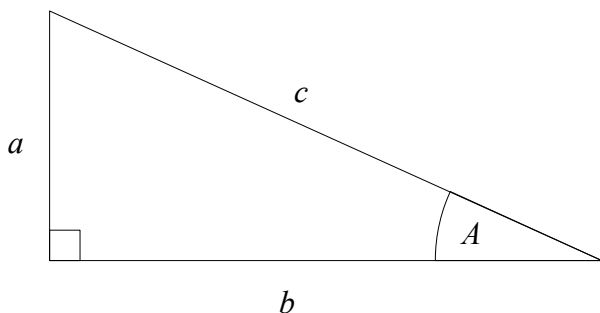
Area of a circle $= \pi r^2$

Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \dfrac{1}{2}ab \sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

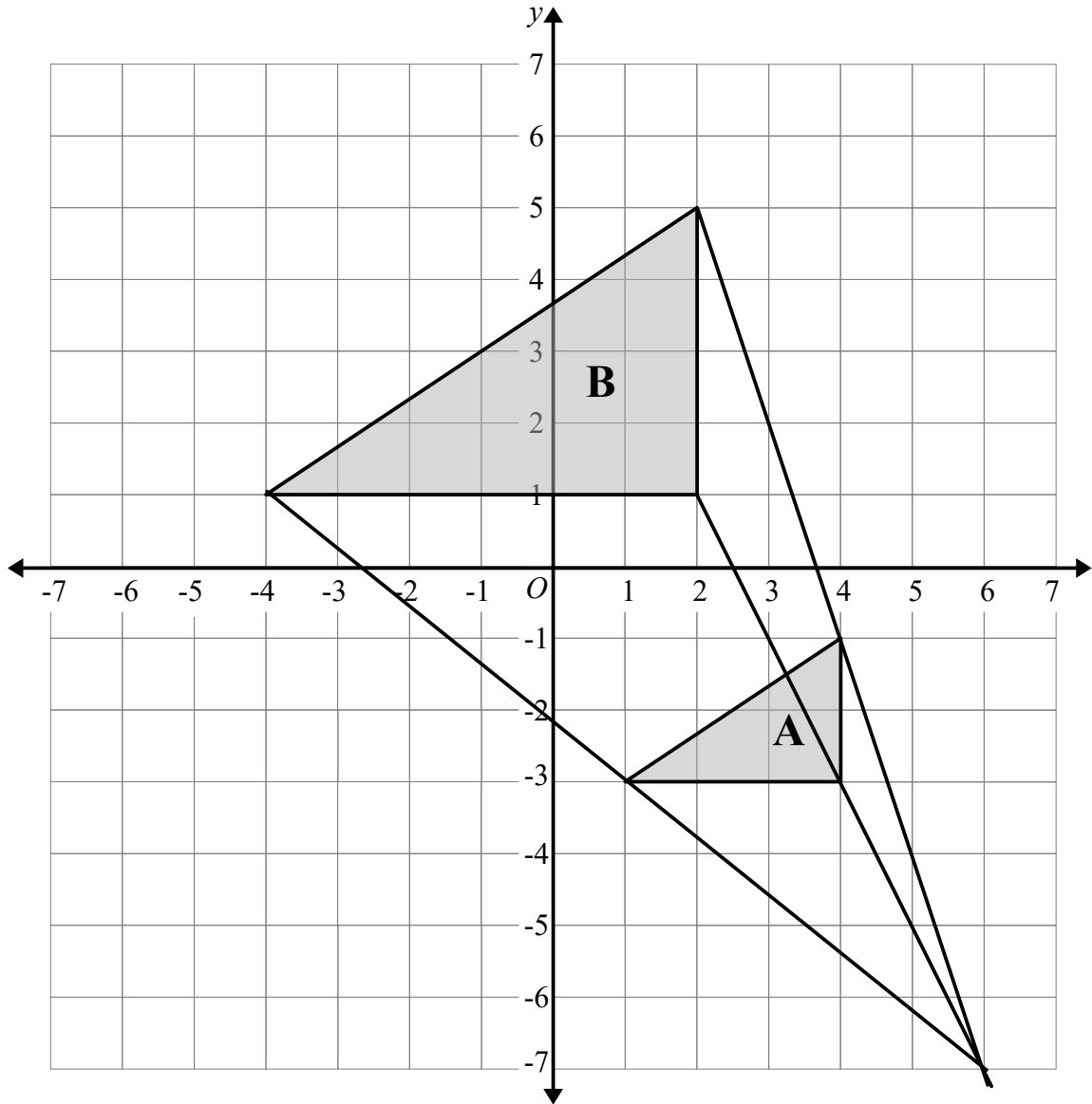Total accrued $= P\left(1 + \dfrac{r}{100}\right)^n$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P $(A$ or $B)$ = P (A) + P (B) − P $(A$ and $B)$

P $(A$ and $B)$ = P $(A$ given $B)$ P(B)

END OF EXAM AID

1

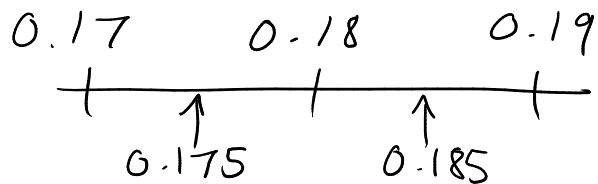Describe fully the single transformation that maps triangle **A** on triangle **B**.

Enlargement, scale factor 2, centre (6,-7)

(Total for Question 1 is 2 marks)

2 A number x is rounded to 2 decimal places.

The result is 0.18

Write down the error interval for x.

$0.17 \qquad 0.18 \qquad 0.19$

$0.175 \qquad 0.185$

$0.175 \leq x < 0.185$

(Total for Question 2 is 2 marks)

3 (a) Simplify $a^9 \times a^4$

a^{13}

(1)

(b) Simplify $(4b^2c)^3$

$4b^2c \times 4b^2c \times 4b^2c$

$64b^6c^3$

(2)

(c) Simplify $d^9 \div d^4$

d^5

(1)

(Total for Question 3 is 4 marks)

4 Given that $\qquad a{:}c = 1{:}6 \qquad$ and $\qquad b{:}c = 2{:}5$

$\times 5 \qquad\qquad \times 6$

Find the ratio $a{:}b{:}c$

Give your answer in its simplest form.

Make Cs the same

$a : c \qquad\qquad b : c$

$5 : 30 \qquad\qquad 12 : 30$

$a : b : c$

$5 : 12 : 30$

$5 : 12 : 30$

(Total for Question 4 is 2 marks)

5   Nick bought a new car.
Each year the car depreciates in value by 12%.

Work out the number of years it takes for the car to half in value.

$$100 \times 0.88^n = 50$$
$$0.88^n = 0.5$$

$$0.88^4 = 0.599\ldots$$
$$0.88^5 = 0.527\ldots$$
$$0.88^6 = 0.4644\ldots$$

(under 50% of value after 6 years)

…...........................6…........years

(Total for Question 5 is 3 marks)

6   In London potatoes cost £0.45 per lb.
In Dublin potatoes cost €1.48 per kilogram.

1 kg = 2.2 lbs
£1 = €1.15

In which city are potatoes better value for money, London or Dublin?
You must show your working.

Dublin :   €1.48 per 2.2 lbs

÷2.2   ÷2.2

€0.672 per lb

÷1.15

£0.58 per lb

London   £0.45 < £0.58

(Total for Question 6 is 3 marks)

7   The diagram shows a patio in the shape of a rectangle.

360cm

3.6 m

9 slabs

4 slabs   1.6 m   160cm
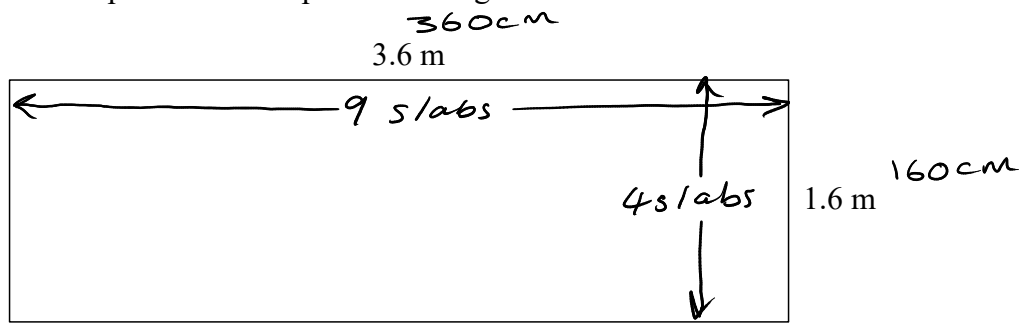
Jack wants to cover the patio with paving slabs. $\frac{360}{40} = 9$ $\frac{160}{40} = 4$
Each paving slab is a square of side 40 cm.

The paving slabs cost £7.59 each.
Jack has £300 to spend on paving slabs.

Does Jack have enough money to cover the patio with paving slabs.

$9 \times 4 = 36$ paving slabs

$36 \times 7.59 = £273.24$

Yes £273.24 < £300

(Total for Question 7 is 4 marks)

8   Use your calculator to work out $\sqrt{\dfrac{\tan 80^\circ + 1}{\tan 80^\circ - 1}}$

(a) Write down all the figures on your calculator display.

1.195051466

(2)

(b) Write your answer to part (a) correct to 3 significant figures.

1.20

(1)

(Total for Question 8 is 3 marks)

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9   Lucy is three times as old as Alex.
Lucy is 7 years older than Megan.
The sum of their ages is 126.

Find the ratio of Alex's age to Lucy's age to Megan's age.

$L = 3A$

$L = M + 7$

$A = \dfrac{L}{3}$ $M = L - 7$

$L + \dfrac{L}{3} + L - 7 = 126$

$\dfrac{7}{3}L - 7 = 126$

$\dfrac{7}{3}L = 133$

$7L = 399$

$\underline{\underline{L = 57}}$

$A = \dfrac{57}{3} = \underline{\underline{19}}$ $M = 57 - 7$
$\qquad\qquad\qquad\quad = \underline{\underline{50}}$ 19 : 57 : 50

(Total for Question 9 is 4 marks)

10 Find the equation of the line parallel to $2x + 5y = 10$ which passes through $(0,-3)$

$$5y = -2x + 10$$
$$y = -\frac{2}{5}x + 2$$

$$M = -\frac{2}{5} \qquad C = -3$$

$$y = -\frac{2}{5}x - 3$$

(Total for Question 10 is 2 marks)

11 Carlos wants to find an estimate for the number of ants in a colony.

He catches 60 ants from the colony and marks each one with a dye.
He then returns the ants to the colony.

A week later, Carlos catches another 60 ants.
8 of these ants are marked with the dye.

(a) Work out an estimate for the number of ants in the colony.

$$\overset{\times 7.5}{\overbrace{\frac{60}{x} = \frac{8}{60}}}_{\times 7.5} \qquad \frac{60}{8} = 7.5$$

$$60 \times 7.5 = 450$$

450

(3)

Carlos assumes that none of the marks had rubbed of.

(b) If Carlos's assumption is wrong, explain what effect this would have on your answer to part (a)

The answer would be too big (overestimate)
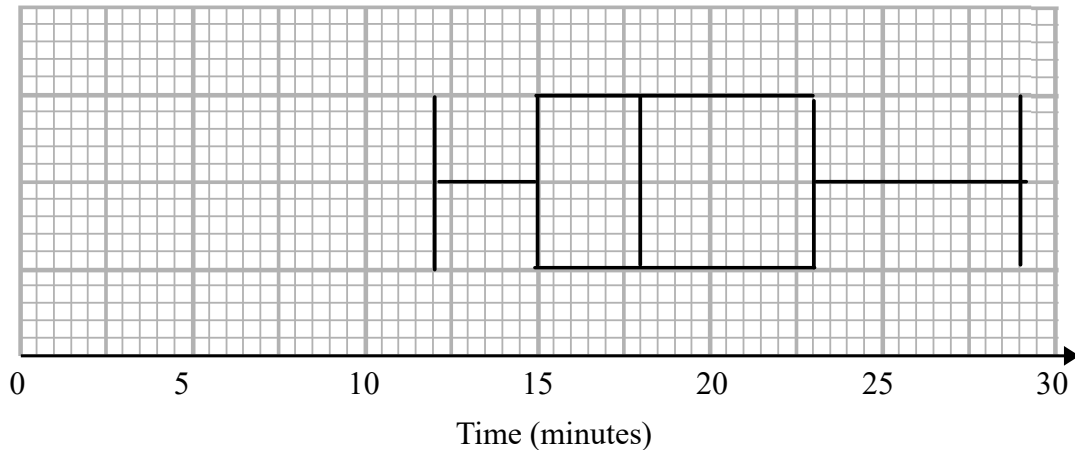
(1)

(Total for Question 11 is 4 marks)

12 The table shows some information about times, in minutes, it took some boys to complete a puzzle.

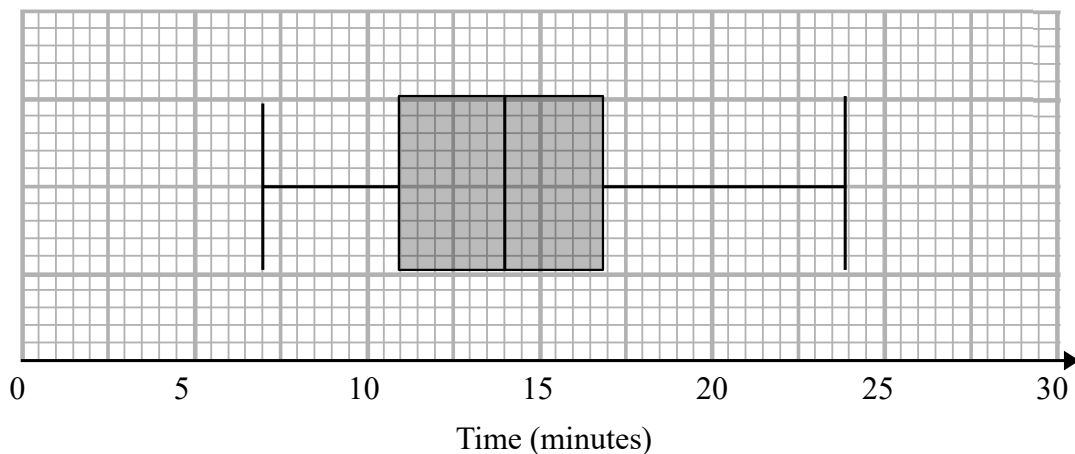| Inter Quartile Range	Minimum	Median	Upper Quartile	Maximum
8	12	18	23	29

(a) Draw a box plot for this information. $LQ = 23 - 8 = 15$

Time (minutes)

(2)

Some girls also completed the puzzle.
The box plot below shows the distribution of times the girls took to complete the puzzle.

Time (minutes)

(b) Compare the distribution of girls' times and the boys' times.

The boy's median time is longer (on average it took the boys longer to complete the puzzle)

The boy's times were more spread out, the boys Inter quartile range is bigger

(2)

(Total for Question 12 is 4 marks)

13 It takes 30 builders 12 days to complete a job.

(a) Work out how many days it would take 40 builders to complete the same job.

$$30 \times 12 = 360 \quad \text{days of work (for 1 builder)}$$

$$\frac{360}{40} = 9$$

9

(2)

(b) Write down any assumption you made in part (a)

All the builders work at the same rate

(1)

(Total for Question 13 is 3 marks)

14 Given that $f(x) = 2x - 4$ and $g(x) = 3x + 5$

(a) Find gf(3)

$$f(3) = 2(3) - 4$$
$$= 2$$
$$g(2) = 3(2) + 5$$
$$= 11$$

11

(2)

(b) Work out an expression for $f^{-1}(x)$

$$f(x) = 2x - 4$$
$$y = 2x - 4$$
$$y + 4 = 2x$$
$$\frac{y + 4}{2} = x$$
$$f^{-1}(x) = \frac{x + 4}{2}$$

$f^{-1}(x) = \frac{x+4}{2}$

(2)

(c) Solve $f(x) = g(x)$

$$2x - 4 = 3x + 5$$
$$-4 = x + 5$$
$$-9 = x$$

$x = -9$

(2)

(Total for Question 14 is 6 marks)

15 The diagram shows a solid shape.
The shape is a hemisphere on top of a cylinder.

45 cm

Volume of a sphere = $\frac{4}{3}\pi r^3$

pressure = $\dfrac{\text{force}}{\text{area}}$
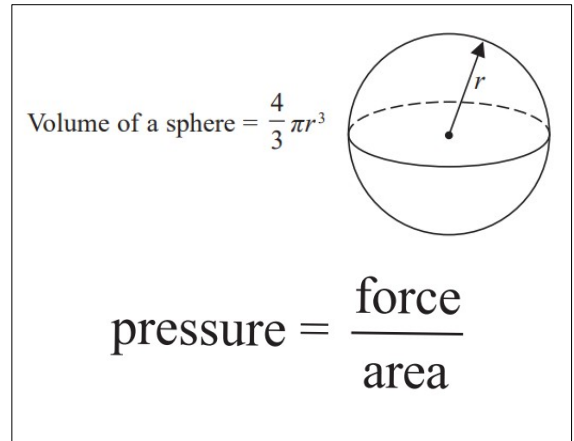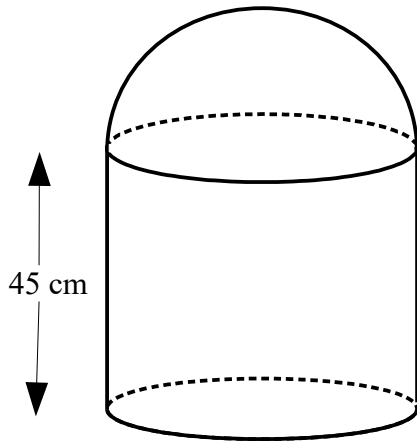
The solid shape is placed on a table.
The solid exerts a force of 4 Newtons on a table.
The pressure on the table is 100 N/m².

The height of the cylinder is 45 cm.

Work out the total volume of the solid shape.

$pressure = \dfrac{force}{area}$

$100 = \dfrac{4}{area}$

$area = \dfrac{4}{100} = \dfrac{1}{25} m^2$

(Area of circle)

$\pi r^2 = \dfrac{1}{25}$

$r^2 = \dfrac{1}{25\pi}$

$r = \sqrt{\dfrac{1}{25\pi}} = 0.1128\ m$

$= 11.28\ cm$

volume of cylinder $= \pi r^2 h$
$= \pi(11.28)^2(45)$
$= 18000\ cm^3$

volume of hemisphere
$= \dfrac{2}{3}\pi r^3$
$= \dfrac{2}{3}\pi(11.28)^3$
$= 3009\ cm^3$

$18000 + 3009 = 21009$

.... 21009 .... cm³

(Total for Question 15 is 6 marks)

16 The diagram shows a rectangle.
All measurements are in cm.

$2x + 7$

$x + 3$
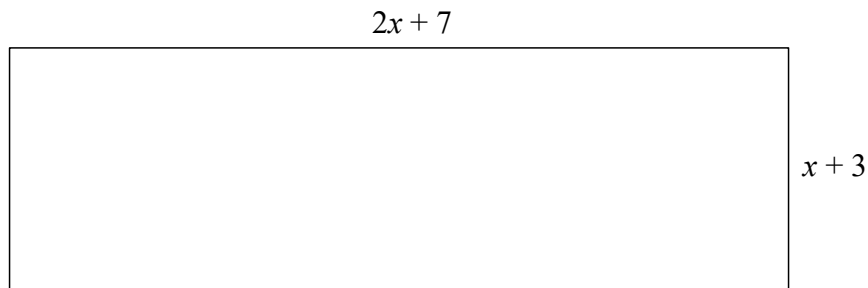
The area of the rectangle is less than 45 cm^2

(a) Show that $2x^2 + 13x - 24 < 0$

$$(2x + 7)(x + 3) < 45$$
$$2x^2 + 6x + 7x + 21 < 45$$
$$2x^2 + 13x - 24 < 0$$

(2)

(b) Find the range of possible values of x.

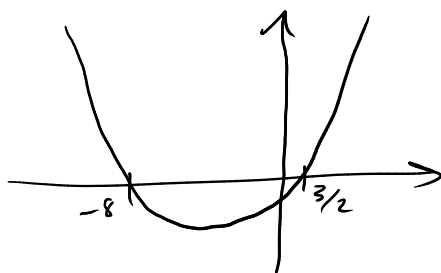$$2x^2 - 3x + 16x - 24$$

$$(x + 8)(2x - 3)$$

$$x = -8 \qquad x = \frac{3}{2}$$

48
1   48
2   24
3   16

$$-8 < x < \frac{3}{2}$$

x also has to be greater than
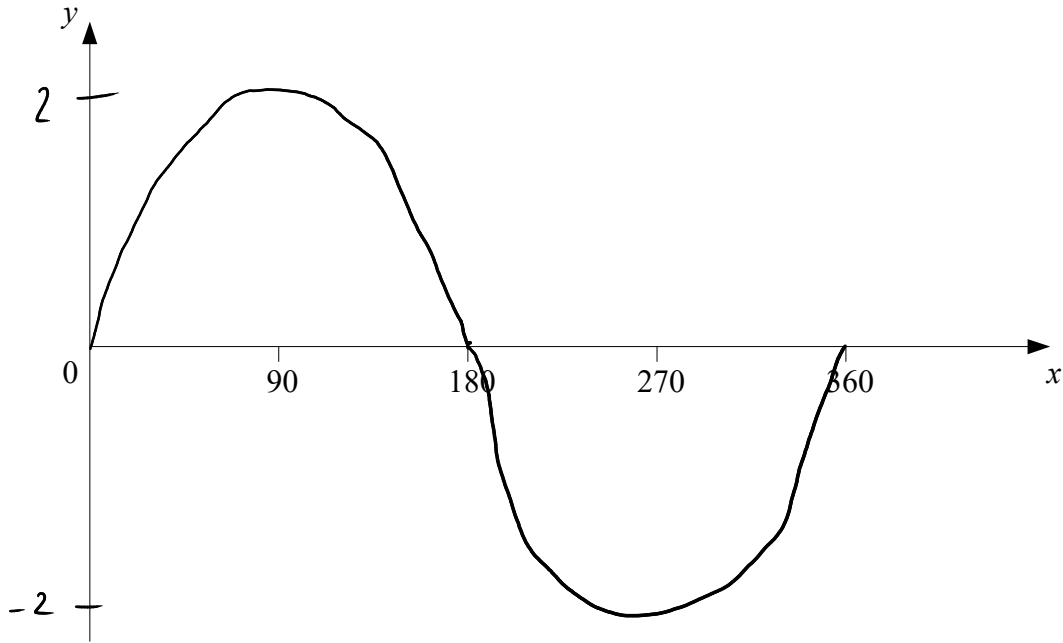-3 or length will be negative
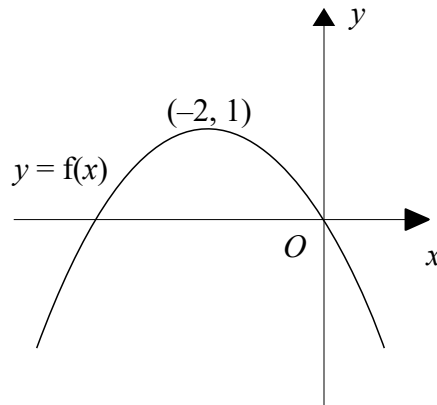
$$-3 < x < \frac{3}{2}$$

(4)

(Total for Question 16 is 6 marks)

17 Sketch the graph of $y = 2 \sin x°$ for $0 \le x \le 360$

(Total for Question 17 is 3 marks)

18 The graph of $y = f(x)$ is shown below.

The coordinates of the maximum point of this curve are $(-2, 1)$.

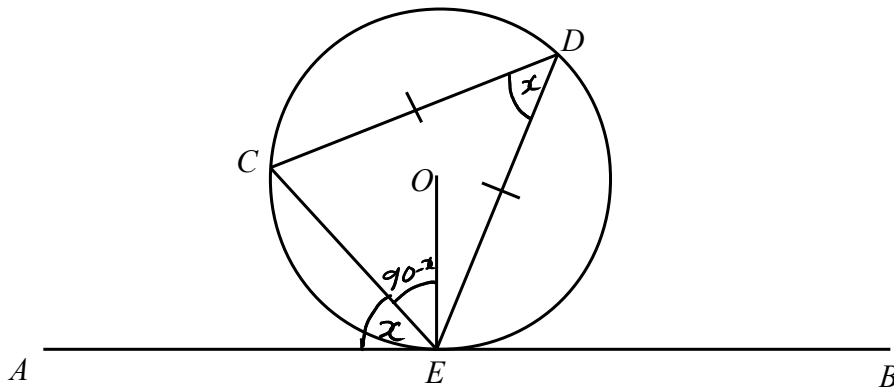Write down the coordinates of the turning point of the curve with equation

(a) $y = f(x) + 3$

$(-2, 4)$

(1)

(b) $y = -f(x + 2)$

$(-4, -1)$

(2)

(Total for Question 18 is 3 marks)

19

C, D and E are points on a circle, centre O.
AEB is a tangent to the circle at E.

CD = DE
Angle AEC = x°

Find the size of angle OED, in terms of x.
Give reasons for each stage of your working.

$CDE = x$ Alternate segment theorem

$AEO = 90$ Tangent meets radius at 90°

$CEO = 90 - x$

$CED = \dfrac{180 - x}{2}$ Angles at the base of an isosceles triangle are equal

$OED = CED - CEO$
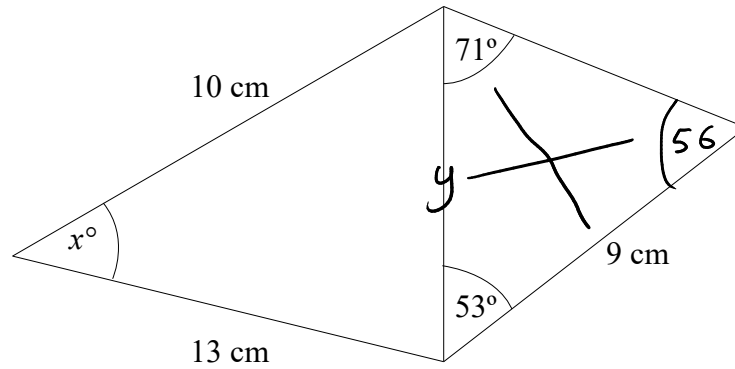
$\quad = \dfrac{180 - x}{2} - (90 - x)$

$\quad = 90 - \dfrac{1}{2}x - 90 + x$

$\quad = \dfrac{1}{2}x$

(Total for Question 19 is 5 marks)

20

71°

10 cm

56

$180 - 71 - 53$
$= 56$

$x°$

9 cm

53°

13 cm

Work out the value of x.
Give your answer to 1 decimal place.

$$\frac{y}{\sin(56)} = \frac{9}{\sin(71)}$$

$$y = \frac{9}{\sin(71)} \times \sin(56)$$

$$= 7.89 \text{ cm}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$a = 7.89$
$b = 10$
$c = 13$

$$\cos A = \frac{(10)^2 + (13)^2 - (7.89)^2}{2(10)(13)}$$

$$\cos A = 0.795$$

$$A = \cos^{-1}(\text{Ans})$$

$$= \underline{37.3°}$$

.....37.3°.....

(Total for Question 20 is 5 marks)

21 **5**0 people were asked which fruits they liked from apples, bananas and oranges.

11 people like all three fruits.
33 people like apples.
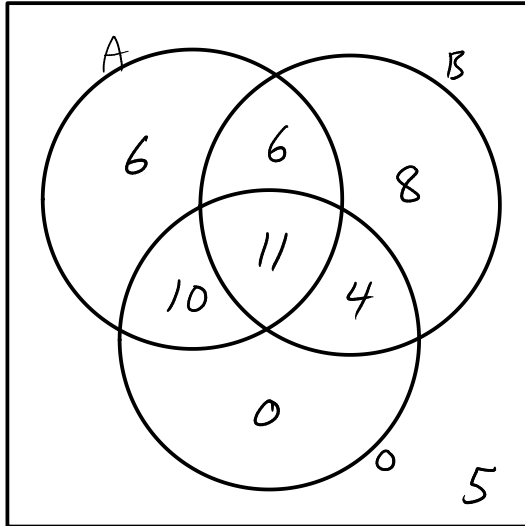6 like apples and bananas but not oranges.
15 like bananas and oranges.
5 of the people do not like any of the fruits.
All 25 people who like oranges like at least one other fruit.

Two of the 50 people are chosen at random.
Work out the probability that they both like bananas.

$$\frac{29}{50} \times \frac{28}{49}$$

$$\frac{58}{175}$$

(Total for Question 21 is 5 marks)