Surname LH

Other name Worked Solutions

Candidate number

Subject

Mathematics

Tier HIGHER

Paper 2H Year 11

23rd February 2022

Time: 1 hour 30 minutes

+10% 1h 39 mins +25% 1h 53 mins

Instructions

- Use **black** ink or ball-point pen.
- Answer **all** questions.
- Answer the questions in the spaces provided
- **Calculators may be used**

Information

- There are 21 questions on this paper
- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Show all of your working out.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium $= \frac{1}{2}(a + b)\,h$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

Circumference of a circle $= 2\pi r = \pi d$

Area of a circle $= \pi r^2$

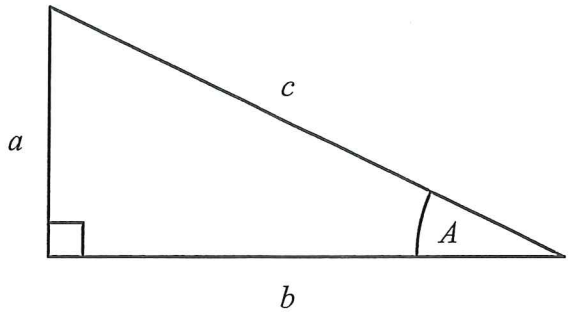
Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

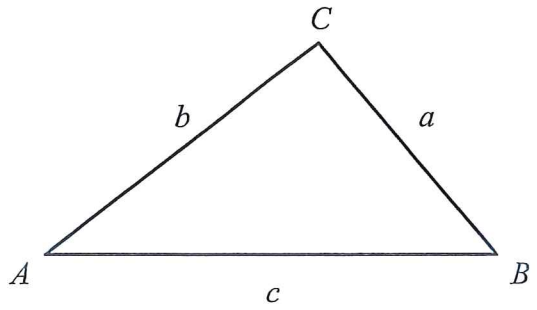


In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b}$$



In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2}\,a\,b \sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

Total accrued $= P\left(1 + \frac{r}{100}\right)^n$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A given B) P(B)

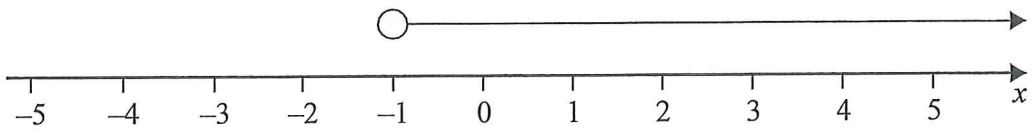
END OF EXAM AID

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

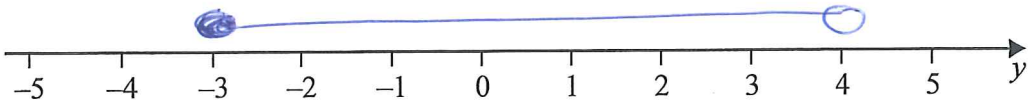
1 (a) Write down the inequality shown on this number line.



$x > -1$

(1)

(b) On the number line below, show the inequality $-3 \leqslant y < 4$



(2)

F tier Q 21

(Total for Question 1 is 3 marks)

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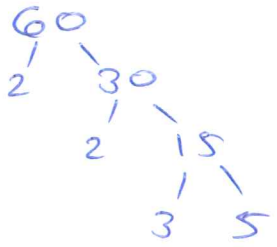
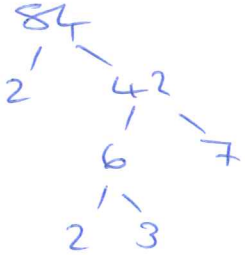
F Tier Q 22

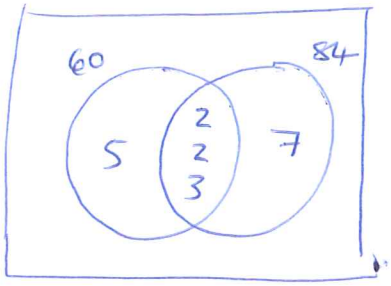
2 (a) Find the Highest Common Factor (HCF) of 60 and 84





$60 = 2 \times 2 \times 3 \times 5$

$84 = 2 \times 2 \times 3 \times 7$

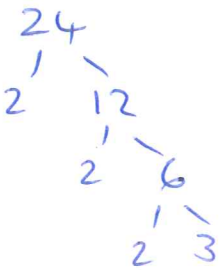


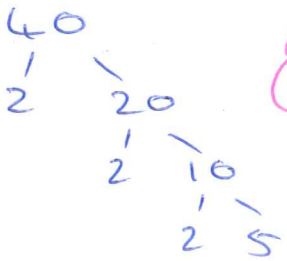
HCF = 2 x 2 x 3

12

(2)

(b) Find the Lowest Common Multiple (LCM) of 24 and 40

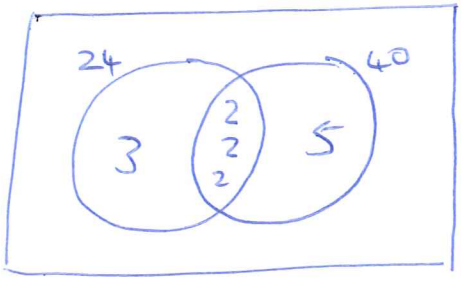




OR
24, 48, 72, 96, 120
40, 80, 120

$24 = 2 \times 2 \times 2 \times 3$

$40 = 2 \times 2 \times 2 \times 5$



LCM
= 3 x 2x2x2 x 5
= 3 x 8 x 5
= 120

120

(2)

(Total for Question 2 is 4 marks)

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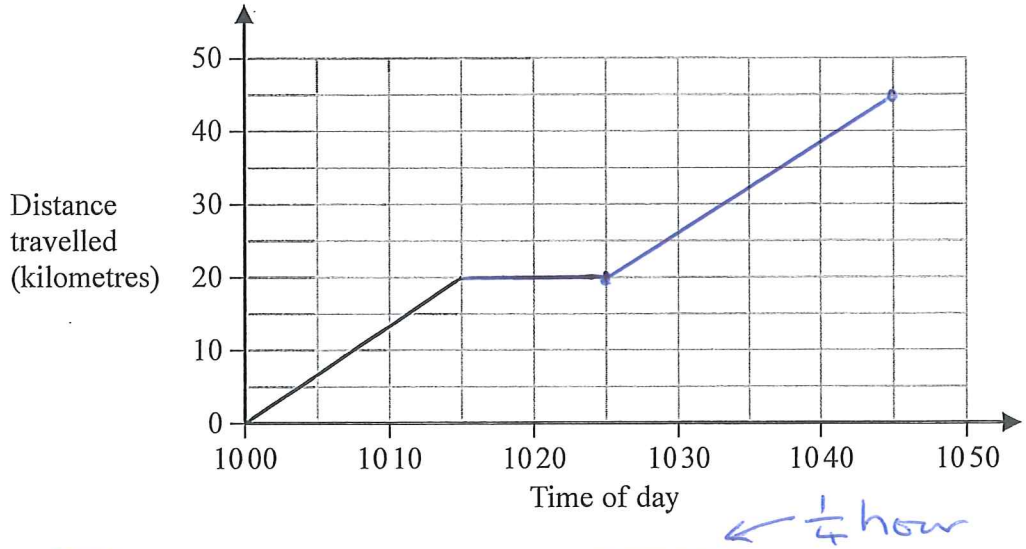
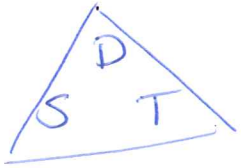
F tier Q 23

3 Sam drives his car on a journey.
Here is the travel graph for the first 15 minutes of his journey.



← $\frac{1}{4}$ hour

(a) Work out Sam's speed, in km/h, for the first 15 minutes of his journey.



20 km : 15 mins
×4 ×4
80 km : 60 mins
80 km/h

OR $S = \frac{D}{T}$

$= \frac{20}{1/4}$

$= 80$

..... km/h

(2)

At 10 15 Sam stops for 10 minutes and then drives for 20 minutes at a speed of 75 km/h.

(b) On the grid, complete the travel graph for Sam's journey.

↑ $\frac{1}{3}$ hour

75 km : 60 minutes
÷3 ↓ ↓ ÷3
25 km : 20 minutes

OR $D = S \times T$

$= 75 \times 1/3$

$= 25$ km

(3)

(Total for Question 3 is 5 marks)

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F Tier Q24

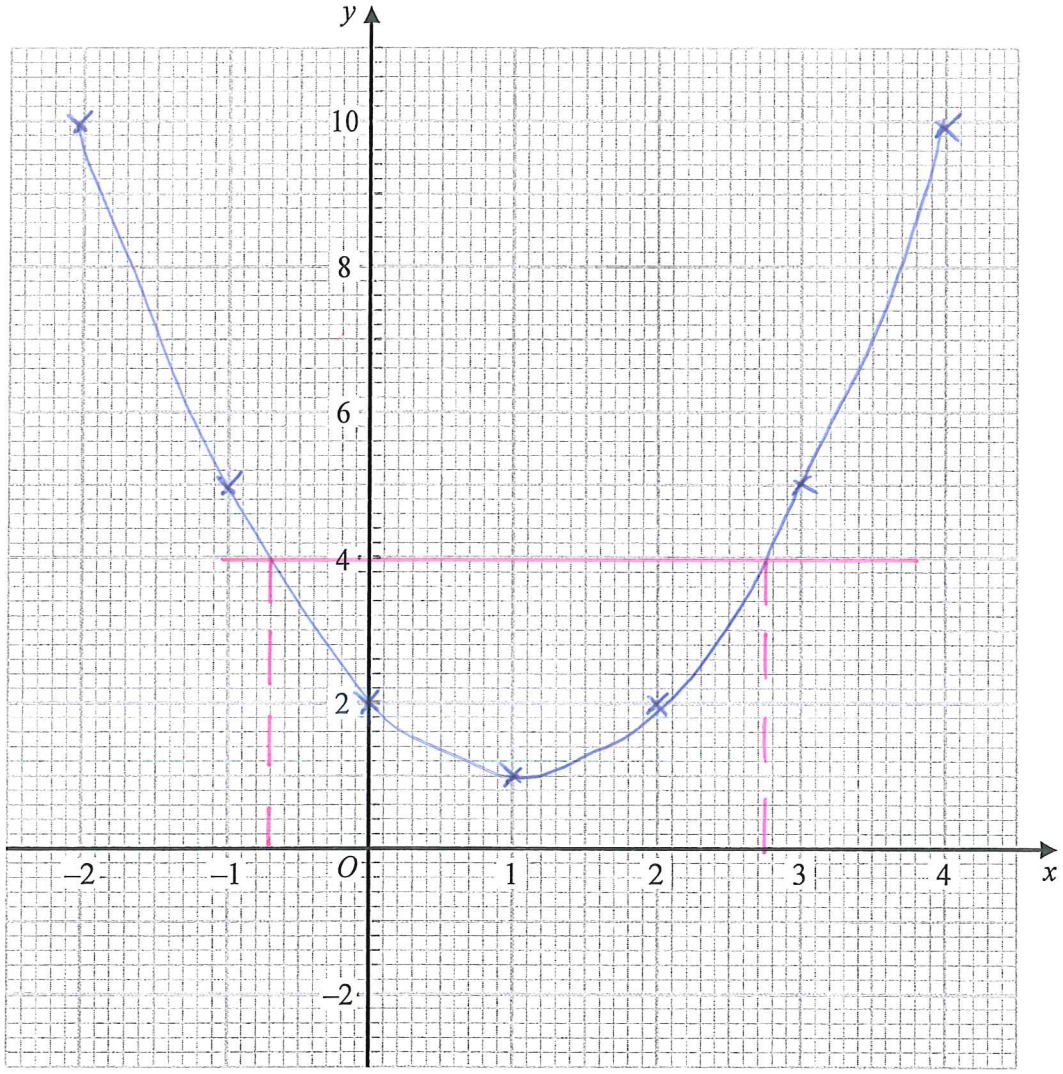
4 (a) Complete the table of values for $y = x^2 - 2x + 2$

x	–2	–1	0	1	2	3	4
y	10	5	2	1	2	5	10

(2)

(b) On the grid, draw the graph of $y = x^2 - 2x + 2$ for values of x from –2 to 4

(2)



(c) Use your graph to find estimates of the solutions of the equation $x^2 - 2x + 2 = 4$

$x = -0.7$ $x = 2.75$

(2)

(Total for Question 4 is 6 marks)

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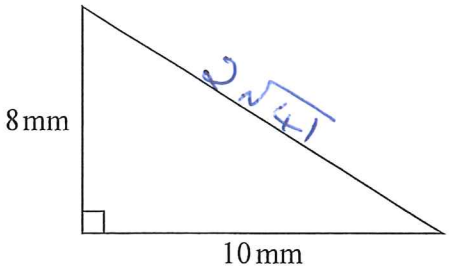
F Q 25

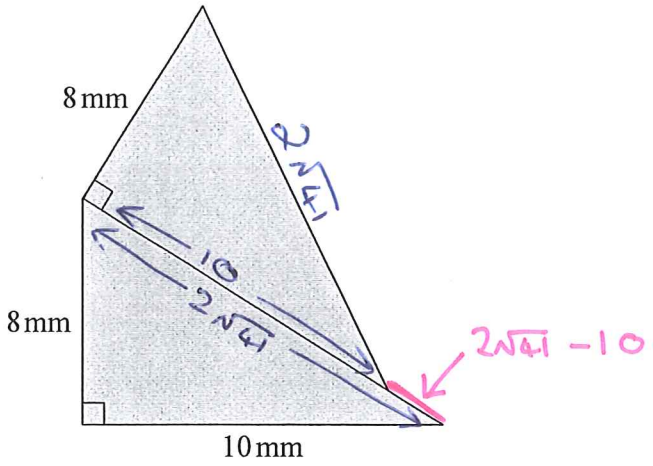
5 Here is a right-angled triangle.

$\sqrt{8^2+10^2}$

$= 2\sqrt{41}$



The shaded shape below is made from two of these triangles.



Work out the perimeter of the shaded shape.
Give your answer correct to 3 significant figures.

Perimeter $= 10 + 8 + 8 + 2\sqrt{41} + 2\sqrt{41} - 10$

$= 16 + 4\sqrt{41}$

$= 41.612496\ldots$

$= 41.6$ mm (3sf)

41.6 ..... mm

(Total for Question 5 is 4 marks)

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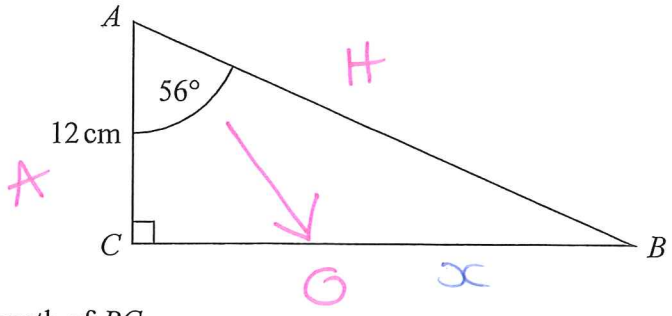
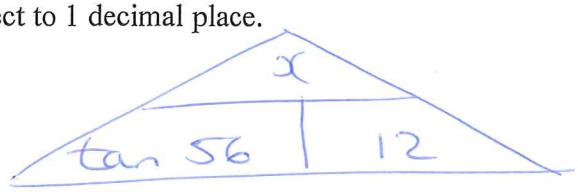
F Tier Q 26

6 *ABC* is a right-angled triangle.



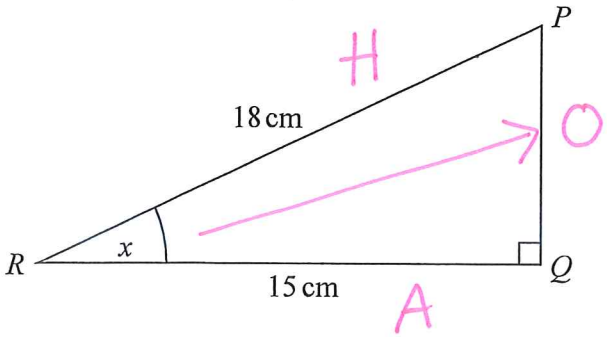
(a) Work out the length of *BC*.
Give your answer correct to 1 decimal place.



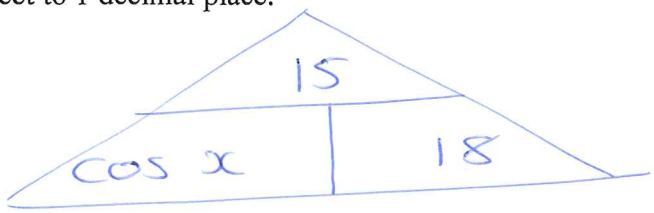
$x = \tan(56) \times 12$
$= 17.79073$
$= 17.8$ cm (1dp)

17.8 cm
(2)

PQR is a right-angled triangle.



(b) Work out the size of the angle marked *x*.
Give your answer correct to 1 decimal place.



$\cos x = \frac{15}{18}$

$x = \cos^{-1}\left(\frac{15}{18}\right) = 33.557309$

33.6 °
(2)

(Total for Question 6 is 4 marks)

Foundation Tier ends here

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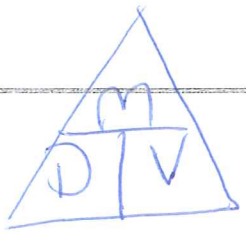

7 Liquid A has a density of 1.8 g/cm³
Liquid B has a density of 1.2 g/cm³

80 cm³ of liquid A is mixed with 40 cm³ of liquid B to make 120 cm³ of liquid C.

Work out the density of liquid C.

Liquid A + Liquid B = Liquid C

D = 1.8 | D = 1.2 | D

M = 144 + M = 48 = M = 192

V = 80 + V = 40 = V = 120

M = D x V
= 1.8 x 80
= 144

M = D x V
= 1.2 x 40
= 48

1.6 g/cm³

(Total for Question 7 is 3 marks)

Remember, density is a compound measure and therefore we cannot add densities.

Density Liquid C

$= \frac{M}{V}$

$= \frac{192}{120}$

$= 1.6$

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8 The grouped frequency table gives information about the time, in minutes, taken by 50 people to solve a puzzle.

Time (t minutes)	Frequency
$0 < t \leqslant 10$	5
$10 < t \leqslant 20$	8
$20 < t \leqslant 30$	12
$30 < t \leqslant 40$	15
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	3

Brian was asked to draw a cumulative frequency table for this information.

This is the table that Brian drew.

Time (t minutes)	Cumulative frequency
$0 < t \leqslant 10$	5
$10 < t \leqslant 20$	13
$20 < t \leqslant 30$	25
$30 < t \leqslant 40$	40
$40 < t \leqslant 50$	47
$50 < t \leqslant 60$	50

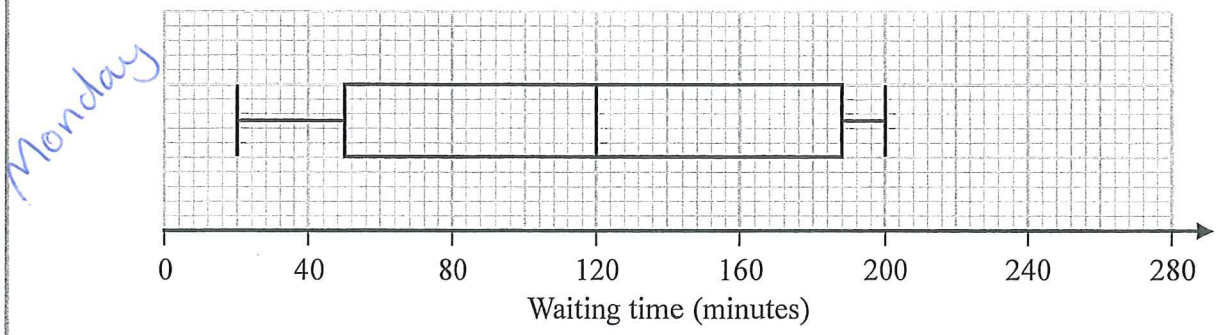
0<t≤20
0<t≤30
0<t≤40
0<t≤50
0<t≤60

Write down **one** thing that is wrong with this cumulative frequency table.

Class widths are for frequency not cumulative frequency

(Total for Question 8 is 1 mark)

9 The box plot shows information about the length of time, in minutes, some people waited to see a doctor at a hospital on Monday.



(a) Work out the interquartile range of the information in the box plot.

IQR = UQ − LQ
= 188 − 50
= 138

138 minutes

(2)

Becky says,

"50% of the people waited for at least 2 hours."

(b) Is Becky correct?
Explain why.

Yes. The median is at 120 minutes (2 hours)
This means 50% of the people waited more than 2 hours

(1)

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The table gives information about the length of time, in minutes, some people waited to see a doctor at the same hospital on Tuesday.

Tuesday

	Waiting time (minutes)
Shortest time	20
Lower quartile	50
Median	100
Upper quartile	140
Longest time	210

Becky was asked to compare the distribution of the lengths of times people waited on Monday with the distribution of the lengths of times people waited on Tuesday.

She wrote,

"People had to wait longer on Tuesday than on Monday."

(c) Give **one** reason why Becky may be wrong.

The median waiting time on Tuesday is 100 minutes which is less than the median waiting time of 120 minutes on Monday.

(1)

(Total for Question 9 is 4 marks)

10 Louise invests £x in Better Investments for 3 years.
Sadiq invests £x in County Bank for 3 years.

Better Investments
Compound Interest
2.5% per annum

County Bank
Compound Interest
2% per annum for the first two years 3.5% per annum for each extra year

At the end of the 3 years, the value of Louise's investment is £344 605

Work out the value of Sadiq's investment at the end of the 3 years.

Louise

$x \xrightarrow{\times 1.025^3}$ £344605

$\div 1.025^3$

original investment

£x = £320000

Sadiq

£320 000 $\xrightarrow{\times 1.02^2}$ $\xrightarrow{\times 1.035}$

£344 580.48

£ 344 580.48

(Total for Question 10 is 4 marks)

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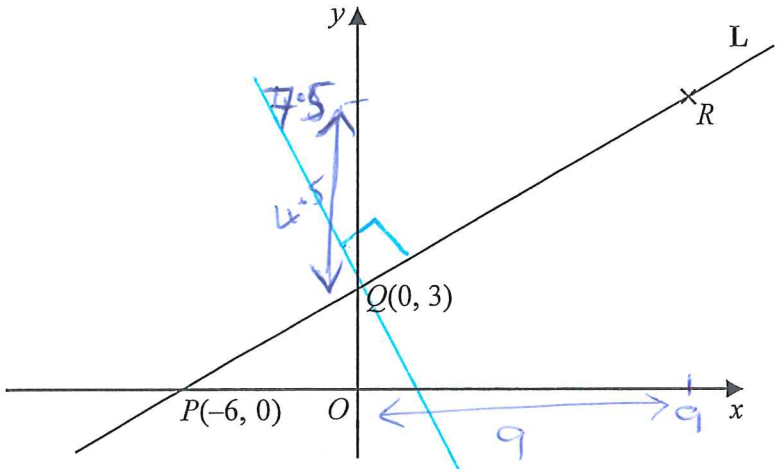
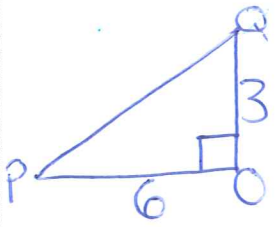
11 Here is a sketch of the line **L**.



The points $P(-6, 0)$ and $Q(0, 3)$ are points on the line **L**.

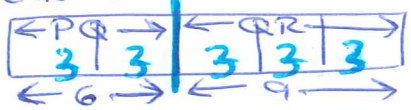
The point R is such that PQR is a straight line and $PQ : QR = 2 : 3$

(a) Find the coordinates of R.

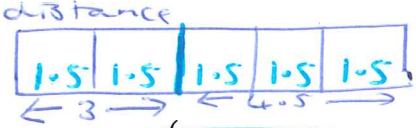


PQ : QR
2 : 3

x distance



y distance



(9, 7·5) (2)

(b) Find an equation of the line that is perpendicular to **L** and passes through Q.

gradient PQ $= \frac{3}{6}$ $\frac{\text{rise}}{\text{run}}$ $= \frac{1}{2}$

so the gradient of L, perpendicular to this, will be -2

equation of L

$y = mx + c$

point (0, 3), gradient -2

y-intercept $c = 3$

$y = -2x + 3$

(3)

(Total for Question 11 is 5 marks)

12 Expand and simplify $(x-2)(3x+2)(2x+3)$

$= (x-2)[6x^2+9x+4x+6]$

$= (x-2)(6x^2+13x+6)$

$= 6x^3+13x^2+6x-12x^2-26x-12$

$= 6x^3+x^2-20x-12$

$6x^3+x^2-20x-12$

(Total for Question 12 is 3 marks)

13 In a school there are 16 teachers and 220 students.
Of these students 120 are girls and 100 are boys.

One teacher, one girl and one boy are going to be chosen to represent the school.

Work out the number of different ways there are to choose one teacher, one girl and one boy.

$16 \times 120 \times 100$

192 000

(Total for Question 13 is 2 marks)

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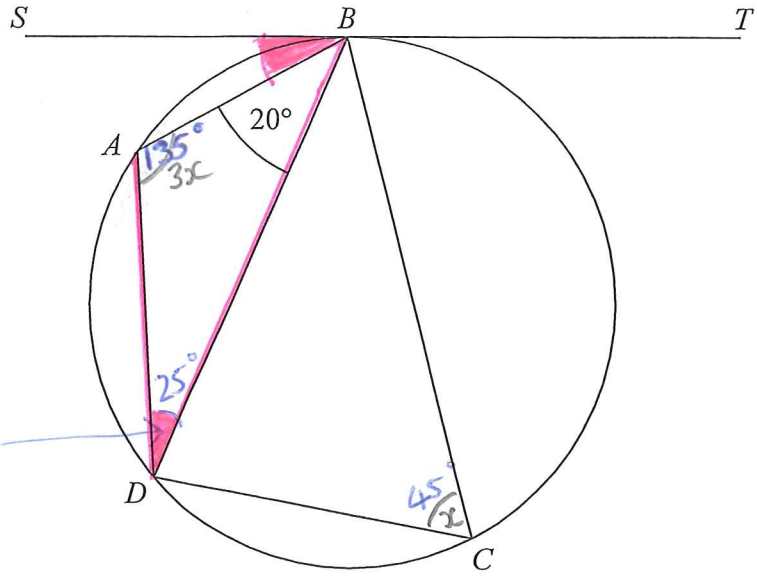
14



angles in a triangle add up to 180°

A, *B*, *C* and *D* are four points on a circle. ABCD is a cyclic quadrilateral
SBT is a tangent to the circle.
Angle *ABD* = 20°

the size of angle *BAD* : the size of angle *BCD* = 3 : 1

Find the size of angle *SBA*.
Give a reason for each stage of your working.

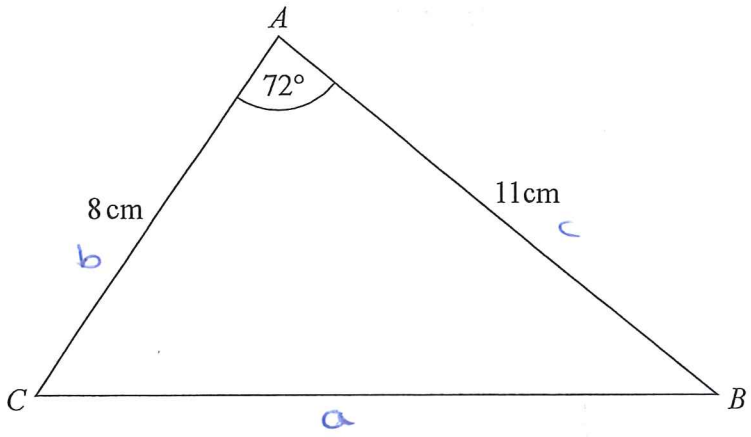
Let $B\hat{A}D = 3x$ and $B\hat{C}D = x$
opposite angles in a quadrilateral add up to 180°
$\therefore 3x + x = 180$
$4x = 180$ [÷ 4
$x = 45°$ angle BCD
$3x = 135°$ angle BAD

angle SBA = 25° alternate segment theorem

25 °

(Total for Question 14 is 4 marks)

15 Here is triangle ABC.



(a) Find the length of BC.
Give your answer correct to 3 significant figures.

"cosy" cosine rule!

cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 72$

$= 130.613009$

$BC = \sqrt{130.613009}$

$= 11.428604\ldots$

$= 11.4$ cm (3sf)

11.4 cm

(3)

(b) Find the area of triangle ABC.
Give your answer correct to 3 significant figures.

Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} bc \sin A$

$= \frac{1}{2} \times 8 \times 11 \times \sin 72$

$= 41.84648\ldots$

$= 41.8\ \text{cm}^2$ (3sf)

41.8 cm^2

(2)

(Total for Question 15 is 5 marks)

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16 (a) Use the iteration formula $x_{n+1} = \sqrt[3]{10 - 2x_n}$ to find the values of x_1, x_2 and x_3
Start with $x_0 = 2$

$x_0 = 2$

$x_1 = \sqrt[3]{10 - 2(2)} = 1.81712\ldots$

$x_2 = 1.85331\ldots$

$x_3 = 1.84626\ldots$

on calculator

2 =

$\sqrt[3]{10 - 2\text{ANS}}$

=

=

=

$x_1 =$ 1.8171

$x_2 =$ 1.8533

$x_3 =$ 1.8463

(3)

The values of x_1, x_2 and x_3 found in part (a) are estimates of the solution of an equation of the form $x^3 + ax + b = 0$ where a and b are integers.

(b) Find the value of a and the value of b.

$x = \sqrt[3]{10 - 2x}$ [cube

$x^3 = 10 - 2x$ [+2x

$x^3 + 2x = 10$ [−10

$x^3 + 2x - 10 = 0$

$a =$ 2

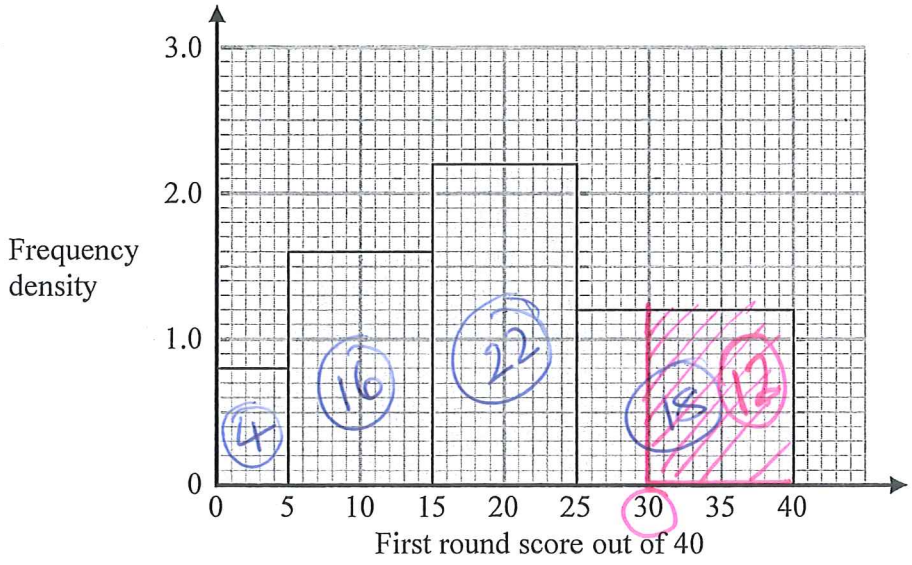
$b =$ −10

(1)

(Total for Question 16 is 4 marks)

17 Some people took part in the first round of a competition.

The histogram gives information about the scores of these people in the first round.



20% of the people got a score high enough for them to qualify for the second round.

Work out an estimate for the score needed to qualify for the second round.
You must show all your working.

first round score	frequency (area of bar)
0 – 5	5 × 0.8 = 4
5 – 15	10 × 1.6 = 16
15 – 25	10 × 2.2 = 22
25 – 40	15 × 1.2 = 18
	60

Total people = 60

20% of 60 = 12

Height of 25 → 40 bar = 1.2

What width would give area of 12?

1.2 × width = 12

width = 10

minimum mark 30

30

(Total for Question 17 is 4 marks)

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18 Here is a graph of $y = \sin x°$ for $0 \leqslant x \leqslant 360$

(a) Using this graph, find estimates of all **four** solutions of

this graph repeats between 360° and 720°

check MS for accepted ranges of answers

$\sin x° = 0.6$ for $0 \leqslant x \leqslant 720$

+360° +360°

36°, 144°, 396°, 504°

(2)

The graph of $y = \sin x°$ is reflected in the x-axis.

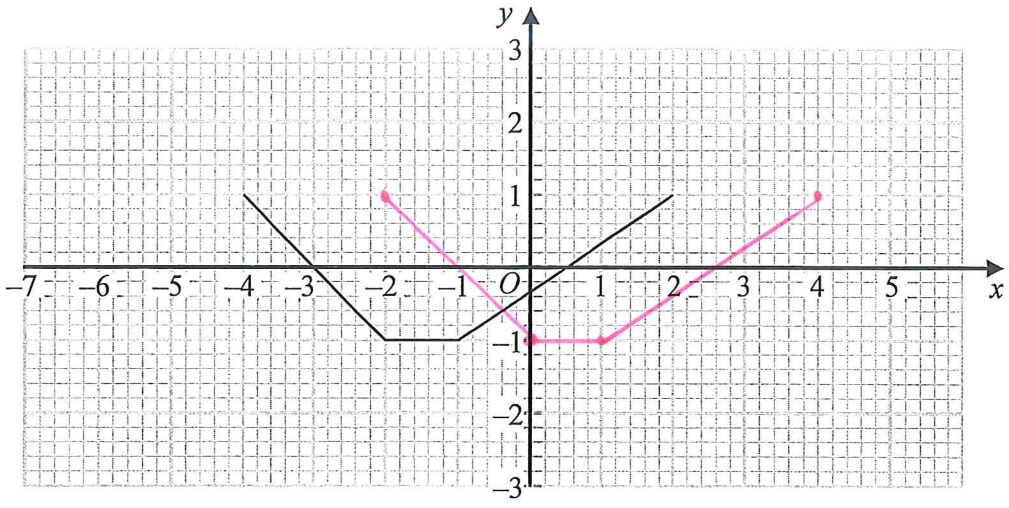
(b) Write down an equation of the reflected graph.

$-f(x)$

$y = -\sin x$

(1)

Here is a graph of $y = f(x)$



(c) On the grid, draw the graph of $y = f(x - 2)$ Right 2

(1)

(Total for Question 18 is 4 marks)

19 A, B and C are three spheres. all spheres will be similar

The volume of sphere **A** is 125 cm^3
The volume of sphere **B** is 27 cm^3

The ratio of the radius of sphere **B** to the radius of sphere **C** is 1 : 2

Work out the ratio of the surface area of sphere **A** to the surface area of sphere **C**.

A B C

$V = 125 \xrightarrow{k^3 = \frac{27}{125}} V = 27$

$k = \frac{3}{5}$ $\qquad$ $k = 2$

$k^2 = \frac{9}{25}$ $\qquad$ $k^2 = 4$

Surface Area A : Surface Area C

$1 \xrightarrow{\times \frac{9}{25} \times 4} \frac{36}{25}$

$\times 25 \downarrow$ $\qquad$ $\downarrow \times 25$

25 : 36

25 : 36

(Total for Question 19 is 3 marks)

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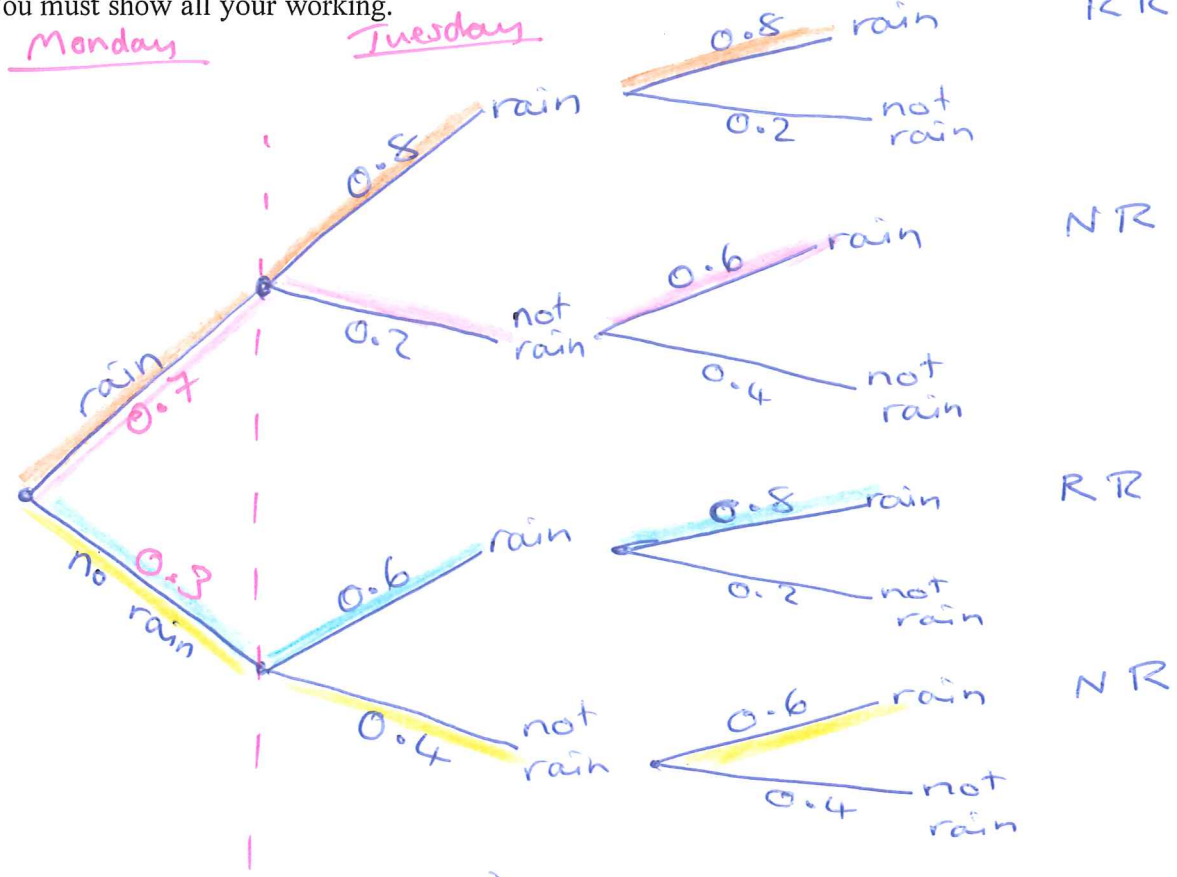
20 In a village,

if it rains on one day, the probability that it will rain on the next day is 0.8
if it does **not** rain on one day, the probability that it will rain on the next day is 0.6

A weather forecaster says,

"There is a 70% chance that it will rain in the village on Monday."

Work out an estimate for the probability that it will rain in the village on Wednesday.
You must show all your working.



P(Rain Wednesday)

$= 0.7 \times 0.8 \times 0.8 + 0.7 \times 0.2 \times 0.6$

$+ 0.3 \times 0.6 \times 0.8 + 0.3 \times 0.4 \times 0.6$

$= \frac{187}{250}$

$= 0.748$

(Total for Question 20 is 4 marks)

21 The time period, T seconds, of a simple pendulum of length l cm is given by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Katie uses a simple pendulum in an experiment to find an estimate for the value of g.

Here are her results.

$l = 52.0$ correct to 3 significant figures.
$T = 1.45$ correct to 3 significant figures.

$51.95 \leq l < 52.05$
$1.445 \leq T < 1.455$

Work out the upper bound and the lower bound for the value of g.
Use $\pi = 3.142$
You must show all your working.

$T = 2\pi\sqrt{\frac{l}{g}}$ [÷ 2π

$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$ [square

$\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g}$

$g = \frac{l}{\left(\frac{T}{2\pi}\right)^2}$

lower bound

$g \approx \frac{51.95}{\left(\frac{1.455}{2\pi}\right)^2}$

$= 968.7669227$

upper bound

$g \approx \frac{52.05}{\left(\frac{1.445}{2\pi}\right)^2}$

$= 984.1125639$

upper bound = 984.1125639

lower bound = 968.7669227

(Total for Question 21 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS

note
these values use π on calculator rather than 3.142
MS accepts values calculated either way.

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